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2. A 50-word summary must accompany the paper.

3. The manuscript (a ribbon copy and two copies) should be double-spaced on one side of 8½-in. by 11-in. paper.

4. The author's full name, Society membership grade, and footnote reference stating present employment must appear on the first page of the paper.

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8. Papers should average about 12,000 words in length and must be no longer than 18,000 words. As an approximation, each full page of typed text, table, or illustration is the equivalent of 300 words.

9. Papers must be written in the third person.

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Proceedings of the American Society of Civil Engineers

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DISCUSSION

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EXPERIENCE WITH A PIER-SUPPORTED BUILDING OVER PERMAFROST

By H. B. Dickens[1] and D. M. Gray[2]

SYNOPSIS

The construction of a two-story steel frame building over permafrost at Churchill, Manitoba, during 1948 to 1950 is described and its performance discussed in relation to the construction techniques used. Its apparent satisfactory performance after nine years of occupancy suggests that the concrete pier and spread footing foundation used can be a practicable alternative to embedded foundations for large heated structures in permafrost regions.

INTRODUCTION

It is only since World War II that serious study has been given in the United States and Canada to the problems of construction in permafrost areas. The wartime construction of the Alaska Highway and the airfields serving the Northwest Staging Route as well as the costly Canol oil pipeline all served to focus attention on this problem and its importance in the future development of Alaska and northern Canada.

As recently as 1943, S. W. Muller, working for the U.S. Army, coined the word "permafrost" during the preparation of his well-known book.[3] Since the

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Note.—Discussion open until March 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SM 5, October, 1960.

1 Div. of Bldg. Research, Natl. Research Council, Ottawa, Canada.

2 Royal Canadian Navy, Dept. of Natl. Defence, Ottawa, Canada.

3 "Permafrost or Permanently Frozen Ground and Related Engineering Problems," by S. W. Muller.

war, planned scientific studies in this field have been carried on in North America by such agencies as the Arctic Construction and Frost Effects Laboratory and the Snow, Ice and Permafrost Research Establishment of the Corps of Engineers in the United States, and by the National Research Council in Canada.

The building described herein was built for the Royal Canadian Navy in the permafrost area of Churchill, Manitoba, during the period 1948 to 1950. At the time of its design and construction little was known in Canada about permafrost and there was no prior Canadian experience of the type that would indicate the suitability of the foundation system proposed for this building at Churchill. Theoretical considerations and an experience reported in the foreign literature were all that were available and although these suggested that the system would be practicable, the project was to a very real extent a pioneering effort in this field.

For this reason, and because the building has now been in use for nearly ten years without any serious performance difficulties, it was thought that this record of its design and construction should be prepared. The authors were not personally connected with the project in any way but they have attempted to reconstruct the details with the aid of the various agencies involved.

THE CHURCHILL AREA

Churchill is located in the north-east corner of the province of Manitoba at at the mouth of the Churchill River on Hudson Bay at latitude 58° 47' N and longitude 94° 11' W. It is approximately 600 air miles north-east of Winnipeg and 510 miles by rail from The Pas. Harbour facilities operated by the National Harbours Board are used chiefly during the short summer season for shipping grain to Europe.

The climate at Churchill is as severe as many locations further north and the area has been used extensively as a cold weather test site by both United States and Canadian test teams. The mean annual temperature is 18.8°F and the mean January daily temperature is -19°F. There are on the average only fifty-nine frost-free days per year. The average annual precipitation is 9.5 in.of rain and 55 in. of snow. The feature that distinguishes Churchill from other cold areas in southern Canada and northern United States is the duration of the cold weather and the combination of wind with low temperature.

The townsite of Churchill is located on the tip of a peninsula between the Churchill River and Hudson Bay. The Department of National Defence maintains a military camp known as Fort Churchill approximately 4 miles to the south-east of the town. Apart from the area of the miliatry camp which is constructed on bedrock lying near the ground surface the foundation conditions in the area are generally poor, being characterized by inadequately drained shallow moss cover over a variety of materials including gravel, silt, and clay interspersed with boulders. Churchill is located at the approximate northern limit of the trees and tree growth in the area is sparse and stunted.

The area is generally considered to lie just inside the zone of continuous permafrost which is defined as that area where permafrost is found everywhere under the natural surface and is relatively thick and massive. Borings taken in 1929 beneath a local lake showed permafrost to begin at 42 ft below

the bottom of the lake. The total thickness of permafrost in the area is not known but earlier records suggest that permafrost may exist to a depth of 230 ft below the surface.

SITE CONDITIONS

The site of the building under review is situated approximately midway between the town of Churchill and the military camp and is located on a low undulating plain characterized by poor natural drainage. It is bounded on the northeast by the road connecting the town with the military camp and on the southwest by the Hudson Bay Railway. Natural drainage in the area is to the southwest towards the Churchill River but is impeded by the railroad embankment

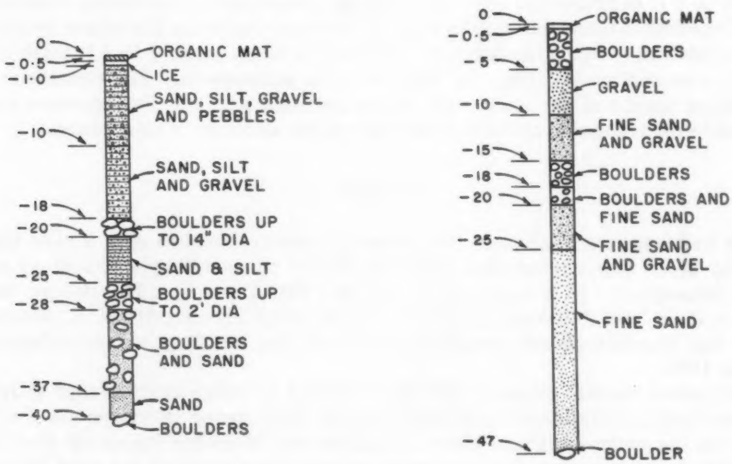

FIG. 1

which is raised 2 1/2 ft above grade. The whole area is covered with a thin mantle of moss, which, owing to the drainage problem, is soggy in the early spring and during the rainy season. The ground normally thaws to depths of 2 1/2 ft to 6 ft in summer, depending on the thickness of moss cover and on the ponding of water.

Soils.—Soil investigations were carried out in the spring of 1948 with a 2 hp diamond drill machine. Typical borehole results obtained by wash borings are shown in Fig. 1. Below the thin top covering of organic matter, the soil consists of sand and silt interspersed with gravel and large boulders. The soil was completely frozen to the maximum depth investigated. All holes had to be stopped at 48 ft because of lack of power in the equipment. The wash borings did not provide details of the ice content of the frozen soil but holes excavated in the area adjacent to the site for the foundations of small ancillary structures showed ice lensing in the soil in the upper few feet of the permafrost zone.

These investigations, combined with other information available from the area, indicated that the subsoils varied in nature across the site, that they were highly susceptible to frost action, and, that upon thawing, would reduce in bearing value and compress under load. This led to the recommendation that the soil at and below the load-carrying level of the foundation must not be allowed to thaw and that care must be taken to reduce the potential effects of frost action in the active layer. Because pile placing was considered too difficult owing to the excessive number of stones and boulders at shallow depths, it was decided to use a foundation design that would eliminate the need for costly subsurface excavation.

A system of concrete piers and spread footings was proposed, to be located at depths of only a few feet below the existing grade and slightly above the original permafrost level. The success of this design was dependent entirely upon the ability to raise the permafrost level above the underside of the footings and to keep it at this level for the life of the structure. This was attempted by placing 4 ft to 5 ft of gravel fill over the existing ground surface following construction of the foundations and by providing an air space between the heated building and the ground. To provide further insulation, a moss layer 1 to 2 ft thick was placed over the gravel fill. As will be seen subsequently, the construction techniques used and the care with which the work was carried out were considered to be critical factors in determining the success of this design.

DESIGN

The building is a T-shaped, two-story structure consisting of a rear wing (48 ft by 96 ft) and a front wing (48 ft by 348 ft) connected by a two-story enclosed passageway 49 ft long and 11 ft wide (Figs 2 and 9). The design was begun in June 1948 following completion of the soils investigations. Construction of the foundations was begun in July 1948; the building was completed in August, 1950.

The superstructure consists of a steel frame to which are attached Q-type insulated wall panels made up of fluted or flat corrugated 16-gauge sheet aluminum on the exterior, glass fibre insulation and 18-gauge flat sheet steel on the interior. In this way, the insulation is kept completely on the cold side of the steel frame and condensation problems are reduced. The exposed steel is covered on the inside with a wallboard material applied over nailable steel studs. Similar studs are used to form the interior partitions. A typical section through the building is shown in Fig. 3. The roof consists of a twenty-year bond smooth surface asphalt and asbestos felt roofing, sloped 3/4 in per ft and applied over 2 in. of rigid insulation with vapor protection carried on a corrugated steel roof deck. The lower or first floor of the building is constructed of a reinforced concrete slab, varying in thickness with differences in loading conditions, placed over a corrugated 18-gauge steel deck which is used as a form and is left in place. From a construction viewpoint, the design was considered to offer the advantage of semi-prefabrication, thus assisting the contractor to close in the building rapidly, an important factor in an area such as Churchill where all labor has to be brought in from the south and where the construction season is shortened by the severe climate.

To facilitate the distribution of services throughout the building, service tunnels are provided for the full length of each section of the building, centrally located beneath the first floor slab. These tunnels contain piping for forced

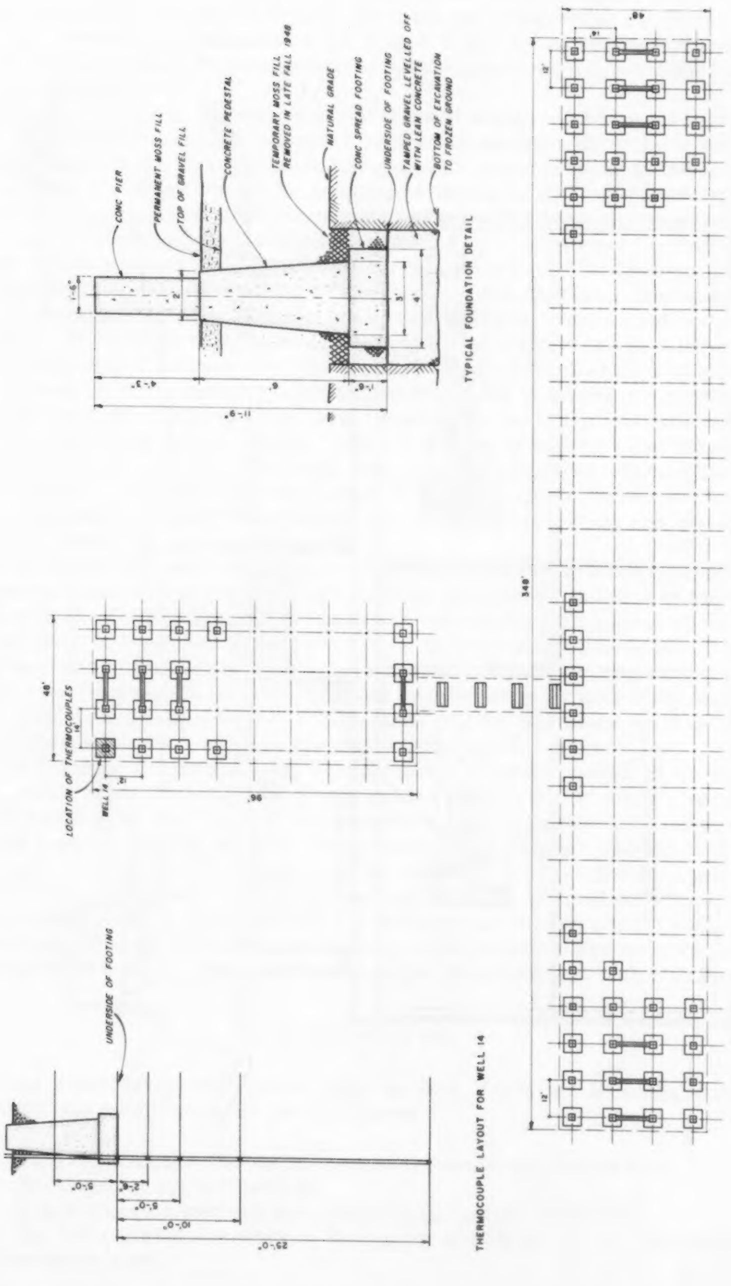
CONC PIER
PERMANENT MOSS FILL
TOP OF GRAVEL FILL
CONCRETE PEDESTAL
TEMPORARY MOSS FILL REMOVED IN LATE FALL 1948
NATURAL GRADE
CONC SPREAD FOOTING
UNDERSIDE OF FOOTING
TAMPED GRAVEL LEVELLED OFF WITH LEAN CONCRETE
BOTTOM OF EXCAVATION TO FROZEN GROUND
TYPICAL FOUNDATION DETAIL
LOCATION OF THERMOCOUPLES
WELL 14
48'
96'
348'
UNDERSIDE OF FOOTING
THERMOCOUPLE LAYOUT FOR WELL 14

FIG. 2

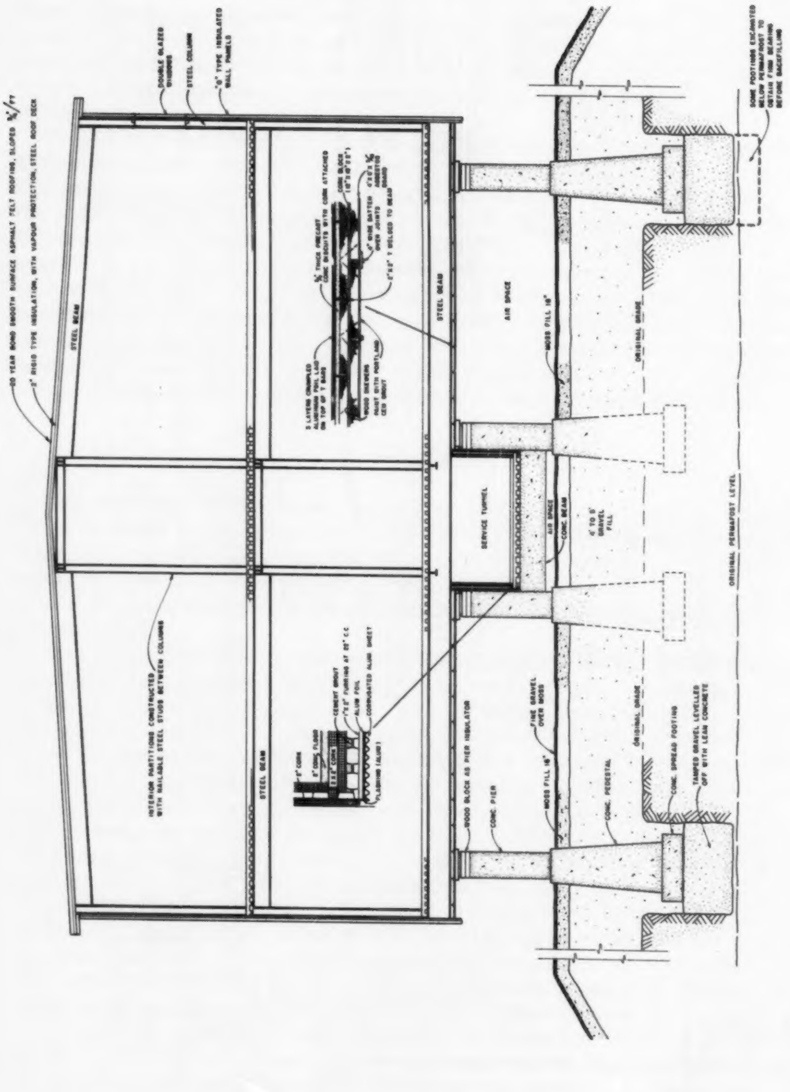
STEEL BEAM
DOUBLE GLAZED WINDOWS
STEEL COLUMN
INTERIOR PARTITIONS CONSTRUCTED WITH NAILABLE STEEL STUDS BETWEEN COLUMNS
SERVICE TUNNEL
AIR SPACE
CONC. BEAM
4' TO 5' GRAVEL FILL
WOOD BLOCK AS PIER INSULATOR
CONC. PIER
MOSS FILL 18"
FINE GRAVEL OVER MOSS
CONC. PEDESTAL
ORIGINAL GRADE
CONC. SPREAD FOOTING
TAMPED GRAVEL LEVELLED OFF WITH LEAN CONCRETE
ORIGINAL PERMAFROST LEVEL
SOME FOOTINGS EXCAVATED BELOW PERMAFROST TO OBTAIN FIRM BEARING BEFORE BACKFILLING

FIG. 3

hot water heating, steam supply and condensate return, domestic hot and cold water supply, fire protection, sewage disposal and compressed air lines. The tunnels measure approximately 8 1/2 ft by 3 1/4 ft in cross-section and are supported on reinforced concrete beams spanning between the two center rows of concrete piers.

Special precautions were taken in the original design to reduce the heat flow from the building to the ground, and thus protect the permafrost, by insulating the underside of the floor of the building and all exposed faces of the service tunnel (Fig. 3). The floor of the building is insulated with aluminum foil and 4 in of cork covered with 3/16-in. asbestos board. The cork was placed in two 2-in layers, the first layer being backed with 3/4 in. of cement plaster to increase its weight and was simply laid in sheets between 2 in-by-2-in. metal tee-bars welded to the bottom flanges of the steel floor beams. Aluminum foil was laid above this cork layer on top of the tee-bars to act as reflective insulation. The second 2-in. layer of cork was cemented to the first layer and also anchored by wood skewers. The undersides of all exposed cork sheets are covered with 3/16-in. asbestos cement board fastened by means of special clips through the cork to the steel tee bars. In addition, all joints are battened.

The floor of the service tunnel consists of 2 in. of concrete applied over 4 in. of cork insulation which is supported on steel panels carried on the concrete beams. The portion of the tunnel floor between the beams is finished with aluminum foil insulation applied over 1-in.-by-2-in. furring and protected by corrugated aluminum sheets.

With downward heat flow, such as occurs through a floor, a large proportion of the heat is transferred by radiation, and thus reflective insulation in the form of aluminum foil is particularly effective in this location. The sides of the tunnel are built of insulated Q-panels 3 1/4 in. in over-all thickness with 2 in. of cork insulation cemented to their interior surface. Heat loss downward through the concrete foundation has been reduced by insulating the tops of all concrete piers with a 16-in.-by-16-by-8 5/8-in. block of glue-laminated wood to break direct thermal contact between the steel beams and the piers.

The building foundation (Fig. 2) consists of concrete piers 18 in. square designed to raise the structure off the ground sufficiently to allow circulation of air between the underside of the first floor and the ground surface and thus reduce heat transfer to the soil. The piers are carried on concrete pedestals 2 ft square at the top, 3 ft square at the base and 6 ft to 7 ft 6 in. high. The sides of the pedestals are sloped to reduce the effect of frost-heaving forces. The building load of each pedestal is transferred to the soil through a 4-ft square concrete spread footing 18 in. deep. The pedestals are anchored to the footings by dowel bars hooked at the ends and extending into both members.

CONSTRUCTION

When construction was begun early in July, 1948, the following detailed schedule was established for the field crew:

1. Provide drainage ditches to remove accumulated surface water.
2. Excavate for spread footings.
3. Place concrete footings and pedestals on tamped gravel fill.
4. Stockpile gravel and moss in the spring of 1949 as fill and insulation for the foundation area.

5. Keep site cleared of snow during the winter of 1948-1949 to allow foundations to freeze in.
6. Before the ground thaws in the spring of 1949, place 4 to 5 ft of gravel fill over the building area and cover with an insulating layer of moss.
7. Place concrete piers and beams to complete substructure.

Although drainage of the site was considered by the design engineers to be of prime importance during the construction phase, insufficient attention was given to this aspect when work began. The first excavations for footings were dug without drainage facilities being provided and by August 15, 1948, the site had become quite wet and the working conditions difficult. A system of temporary drainage ditches 2 ft wide and a minimum of 2 ft 9 in. deep were then dug. These were located between foundation bays to avoid undermining the footings when they were placed. The ditches were excavated to frozen ground and in some cases, to obtain proper grade, were taken to slightly below the level of permafrost. The temporary ditches were led into a permanent ditch excavated around the perimeter of the building. This ditch drained to the low area behind the building through culverts constructed under the railway embankment. The top 3 ft of soil dried out rapidly after this ditch was completed and permitted construction to proceed with much less difficulty. For the remainder of the work, care was taken to keep disturbance of the ground to a minimum. Instructions were given to the construction crew to remove moss cover and top soil only where necessary for placing footings.

The elevation of the natural grade varied from 42 ft to 44 ft above mean sea level across the site and that of permafrost from 35 ft to 40 ft. The undersides of the footings were designed to be placed at elevations of 40 ft and 41 ft 6 in. It was intended to excavate to the seasonal frost level and to backfill to the footing level with tamped gravel. When footing excavations were carried to this level, however, some were not completed quickly enough and the soil in them became very soft. In such cases it was necessary to continue the excavations to permafrost or deeper to obtain firm bearing and then to backfill to the footing level with mechanically tamped gravel. This occurred particularly with the foundation for the rear wing where a 6-ft-by-6-in gravel fill was placed under each 4-ft-by-4-ft footing to distribute the heavier floor loads in that section of the building. The final depth of gravel under each footing varied from 2 ft to 2 ft 9 in. in the front wing and from 4 ft to 7 ft in the rear wing.

Before placing the concrete spread footings, the top of the gravel fill was levelled off with a thin layer of lean concrete. By September 27, 1948 all footings were completed and by October 7 all concrete pedestals were in place. The pedestals were large enough at the top where the piers joined them to allow for some adjustment if misalinement occurred before the concrete piers were constructed. Level observations taken on top of the pedestals when first placed, again when closing the job for the winter, and finally when opening it again in the spring, showed that no significant movements had occurred and such adjustment was not needed. Effective drainage of the site doubtless did much to keep foundation movement to a minimum.

As soon as the concrete work permitted, gravel backfill was placed around the footings and covered with a temporary moss fill to protect the permafrost during the remainder of the warm season. This moss fill was removed late in 1948, when the air temperatures were below freezing, by which time the natural moss cover over the site had also been removed and piled ready for use as fill

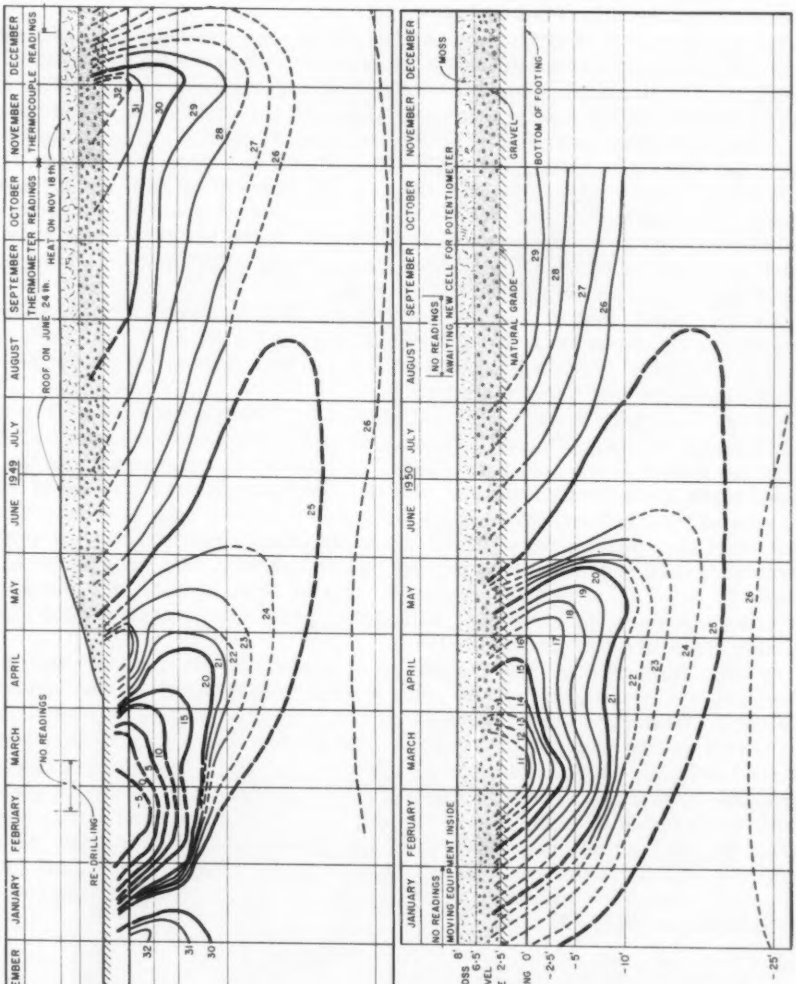
DECEMBER JANUARY FEBRUARY MARCH APRIL MAY JUNE 1949 JULY AUGUST SEPTEMBER OCTOBER NOVEMBER DECEMBER
NO READINGS
RE-DRILLING
ROOF ON JUNE 24 th.
THERMOMETER READINGS
HEAT ON NOV 18 th.
THERMOCOUPLE READINGS
MOSS 8'
6·5'
GRAVEL
NATURAL GRADE 2·5'
BOTTOM OF FOOTING 0'
-2·5'
-5'
-10'
-25'
JANUARY FEBRUARY MARCH APRIL MAY JUNE 1950 JULY AUGUST SEPTEMBER OCTOBER NOVEMBER DECEMBER
NO READINGS
MOVING EQUIPMENT INSIDE
NO READINGS
AWAITING NEW CELL FOR POTENTIOMETER
MOSS
GRAVEL
NATURAL GRADE
BOTTOM OF FOOTING

FIG. 4

insulation the following year. Eight thousand to ten thousand cu yd of gravel were also stockpiled near the building and all temporary ditches were back-filled with coarse gravel before freeze-up.

During the winter of 1948-1949, the foundation area was kept clear of snow to permit maximum possible cooling of the ground over the building site (Fig. 5). Note the battered pedestals and reinforcing for concrete piers shown in Fig. 5. Some indication of the cooling achieved during this period is given by the temperature isotherms shown in Fig. 4. The temperature is in F°. Although snowfall in the Churchill area is not high, snow drifting is a problem owing to the frequency of winter winds. To assist in keeping snow off the site, no obstructions such as worksheds or stockpiles of materials which would lead to formation of snow drifts, were permitted to windward of the building area. Snow, which accumulated between the foundations, was removed during the winter by a combination of mechanical and had labor methods.

Early in April, 1949 before the ground began to thaw, gravel stockpiled the year before was used to place a 4 ft to 5 ft depth of fill over the site to a distance of 20 ft outside the building area. This fill was placed between the pedestals to a height just below their tops.

To avoid trapping water in the fill which would increase the possibility of frost heaving in future winters, 3-in. diam slotted fibre pipes were sunk to 1 ft below the bottom of the gravel pads supporting the footings to serve as sumps. These pipes were located midway between alternate pedestals in each of the two central footing lines. Wood plugs were placed in their tops to reduce circulation of air. Surplus water draining into the pipes through 1/2-in. by 4-in. slots cut in their sides could be pumped out periodically, thus keeping the fill relatively dry. As a further precaution against frost heaving, the sides of the pedestals were thickly coated with heavy grease and were wrapped with building paper prior to placing the fill.

By the middle of May 1949, the placing of gravel fill was complete and the moss that had been stockpiled was being spread over the gravel as an insulation layer. Meanwhile, work had begun on the formwork for the concrete piers and beams and by the end of May the concrete work on the substructure was finished (Fig. 6). In Fig. 6 note the gravel fill and moss layer placed to the top of the pedestals and the concrete piers constructed. The structural steel frame was erected in June (Fig. 7) and the building was advanced sufficiently during the summer to permit inside work during the winter of 1949-1950. Note the gravel fill and moss under the building in Fig. 7. The service tunnel and air space beneath the building can be seen in Fig. 8. The building was completed in August 1950 (Fig. 9).

PERFORMANCE

Although the building was not the subject of a research study ground temperature measuring instruments were installed at several points under the structure. This was done in the interests of verifying the design assumption that the permafrost would be raised to the footing level and would remain there during operation of the building.

Thermocouple strings placed in temperature wells drilled by a diamond drill machine were installed at twelve different footing locations chosen to reflect the varying influence of such factors as heat loss from the building and exposure to the sun. Two additional wells were located outside the building area for

FIG. 5

FIG. 6

FIG. 7

FIG. 8

reference purposes. Thermocouple measurements were begun on November 10, 1949 at a time when the foundations, gravel fill, and insulating moss cover were in place and the building had been closed in to permit work to continue inside during the winter months. These measurements were taken at 10-day intervals until November 20, 1950 with occasional interruptions because of instrument difficulties.

Prior to these thermocouple observations, measurements were made for a 12-month period in three of the temperature wells using mercury thermometers. In one of these, well No. 14, temperatures were recorded to depths of 8

FIG. 9

ft below the footing from December, 1948 to February, 1949, and to 25 ft below the footing from March to October, 1949, at which time the thermocouples were installed.

The location of well No. 14 is shown on the foundation plan (Fig. 2), together with details of its thermocouple arrangement. This well is under the rear of wing of the building in an area where the water condition during construction in the summer of 1948 was particularly bad.

An isotherm plot of the temperatures recorded in well No. 14 is given in Fig. 4. The observations were not continued for a sufficiently long period to

be conclusive and were subject to instrument and observer error. They do provide some information and indicate certain trends as follows:

1. The intensive cooling of the ground in the winter of 1948-1949 as indicated by the close spacing of the 5° isotherms shows the value of keeping the site cleared of snow during that period.
2. The mean ground temperature at a depth of 30 ft is approximately 26°F.
3. The maximum temperature of the ground at the footing level is not reached until sometime in November.
4. Despite the erratic nature of some of the observations, the temperatures suggest that the soil at the footing level did remain frozen during the summer of 1949 and again in 1950, after the building had been heated for one winter.

Ground temperature measurements unfortunately were discontinued after November, 1950 and the thermocouple installations have since been either removed or made unserviceable. During the ensuing 9-yr period that the building has been in use, however, there has been no indication of foundation movement. Maintenance in this period has been limited to refastening of the cork insulation and asbestos board attached to the underside of the first floor, some of which became loose.

CONCLUSIONS

The apparent satisfactory performance after nine years of occupancy of this pier-supported building at Churchill indicates that the design and construction techniques used have been successful in combatting the problems of permafrost in that area.

Detailed research studies were not carried out but this review of the construction and performance of the building suggests that the foundation system used can be a practicable alternative to foundations embedded in permafrost for large heated structures in northern regions, providing that careful attention is given to field procedures which aid in establishing and maintaining a satisfactory ground thermal regime beneath the building.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the assistance of Per Hall, Vice-President, and J. N. Galli, Project Engineer, Foundation of Canada Engineering Corporation Limited, in providing information for this paper. Mr. Hall was directly concerned with the design and construction of this building and has supplied valuable information based on his own personal recollection of the work. In addition, he has generously made available all company records on this project. Mr. Galli assisted in selecting appropriate material from the company's records and kindly arranged for discussions with Mr. Mathys and Mr. Johnson, soils engineers and driller, respectively, on this project.

Figs. 1 to 4 are based on drawings supplied by the Foundation of Canada Engineering Corporation Limited.

This joint paper by the Division of Building Research, National Research Council and the Department of National Defence is presented with the approval Mr. R. F. Legger, Director, Division of Building Research, and Capt. J. B. Roper, Civil Engineer-in-Chief, Naval Technical Services, Royal Canadian Navy.

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SEEPAGE REQUIREMENTS OF FILTERS AND PERVIOUS BASES

By Harry R. Cedergren,[1] M.ASCE

SYNOPSIS

The water-removing capabilities of two common but distinctly different types of filter designs are analyzed by the flow-net. Typical solutions and numerical examples are presented to emphasize the importance of boundary conditions and permeability upon water-removing capacity.

INTRODUCTION

A rational method is available[2] for the design of base courses that will drain rapidly after becoming flooded by entry of water from the sides, through the pavement, by temporary rises in ground water level, and so forth, and the influence of capillarity can be estimated.[3,4] Up to the present time, however, there has been no rational method for predicting minimum design requirements of filters and pervious bases that will assure the continued removal of infiltering ground water or other steady seepage without excessive build-up of hydrostatic head.

The water-removing capacity of pervious filters and bases varies with boundary conditions, permeability, and the thickness of the water-removing layer. In some types of installation sufficient water-removing capacity is easily attained; however, in others, the discharging seepage is removed with

Note.—Discussion open until March 1, 1961. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SM 5, October, 1960.

1 Sr. Materials and Research Engr., Calif. Div. of Highways, Sacramento, Calif.

2 "Base Course Drainage for Airport Pavements," by A. Casagrande and W. L. Shannon, Proceedings, ASCE, Vol. 77, June, 1951, pp. 792-820.

3 "Subsurface Drainage of Highways and Airports," Bulletin 209, Highway Research Board, 1959, pp. 3-6.

4 "Procedures for Testing Soils," American Society for Testing Materials, September, 1944, pp. 56 and 58.

sufficient rapidity only through careful control of the key factors. Flow-net solutions are presented to illustrate a rational approach to filter design and to show the relative importance of some basic factors.

FILTERS MUST SATISFY CONFLICTING REQUIREMENTS

The safe, quick removal of seeping water from engineering structures constructed of earth or founded on earth is often an essential design requirement. Earth dams and levees usually are provided with drains and filters for the removal of seepage caused by the hydraulic conditions that they create (Fig. 1(b)). Airport and highway pavements and roadbeds frequently are protected from the harmful effects of surface infiltration and ground water intrusion by "pervious" bases or filter blankets (Fig. 2). Without adequate drainage many projects would not be economically feasible. In fact, drainage facilities have made many projects possible.

The remarkable benefits possible from filters can be fully assured only if their gradation is carefully adjusted to meet the requirements of the job. Specifications for the grading of granular materials for filters and pervious bases are established by the need for satisfying two conflicting requirements. It must insure permanence of operation. Filters or other porous water-removing layers must be properly graded to prevent the movement of soil particles into or through the pores of the filter. However, it must also permit the rapid removal of water. Filters must be sufficiently more pervious than the protected soil so that the incoming water will be removed without appreciable build-up of hydrostatic head in the filter.

Criteria that have been used since about 1930 for the design of filters for dams and levees, and have been referred at as "Terzaghi Criteria," may be stated as follows:

1. The 15% size of the filter material, D_{15}, must be not more than 4 or 5 times the 85% size, D_{85}, of the protected soil (to prevent piping).
2. The 15% size of the filter material, D_{15}, must be at least 4 or 5 times the 15% size of the protected soil, D_{15}, (to insure adequate permeability).

Experimental work and field observations by a number of investigators[5,6,7,8] has verified the general suitability of the above criteria; however, some modifications have been proposed. Generally the stipulation is made that the grain-size curves of soil and filter shall be somewhat parallel.

If a filter is allowed to be too coarse with respect to the protected soil, and severe discharge conditions develop, serious internal erosion and piping may be expected.

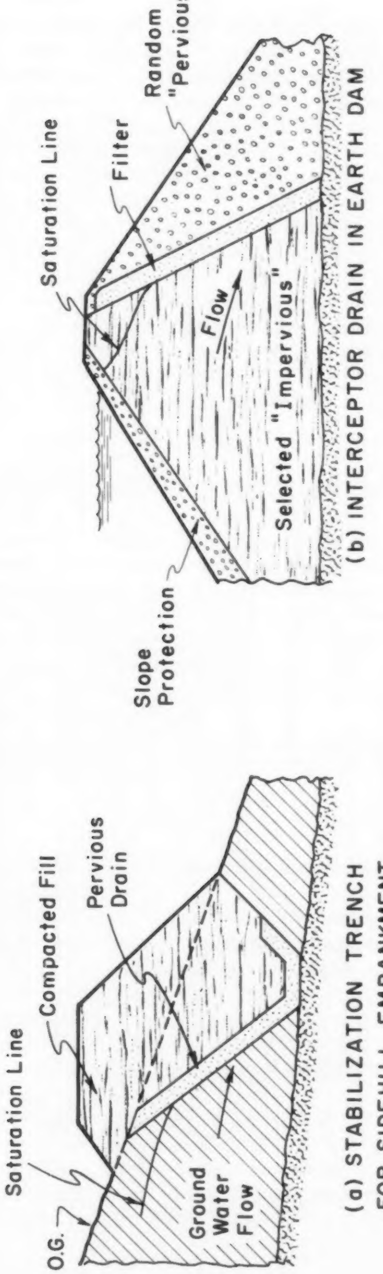
Criterion No. 2 generally has been assumed to assure sufficient permeability to hold seepage forces and gradients in filters to small amounts. It does not

5 "An Experimental Investigation of Protective Filters," by G. E. Bertram, Harvard Graduate School of Engineering, Pub. 267, Series 7, 1940.

6 "Design of Drainage Facilities for Airfields," Office of Chief of Engineers, War Department, Engrg. Manual, February, 1943, Ch. XXI.

7 "The Use of Laboratory Tests to Develop Design Criteria for Protective Filters," by K. P. Karpoff, Proceedings, ASTM, Vol. 55, 1955, pp. 1183-1193.

8 "Underdrain Practice of the Connecticut Highway Department," by Philip Keene, Proceedings, Highway Research Board, 1944, pp. 377-389.



(a) STABILIZATION TRENCH FOR SIDEHILL EMBANKMENT

(b) INTERCEPTOR DRAIN IN EARTH DAM

FIG. 1.—EXAMPLES OF ENGINEERING DESIGNS UTILIZING SLOPING PERVIOUS FILTERS OR DRAINS FOR REMOVAL OF STEADY SEEPAGE

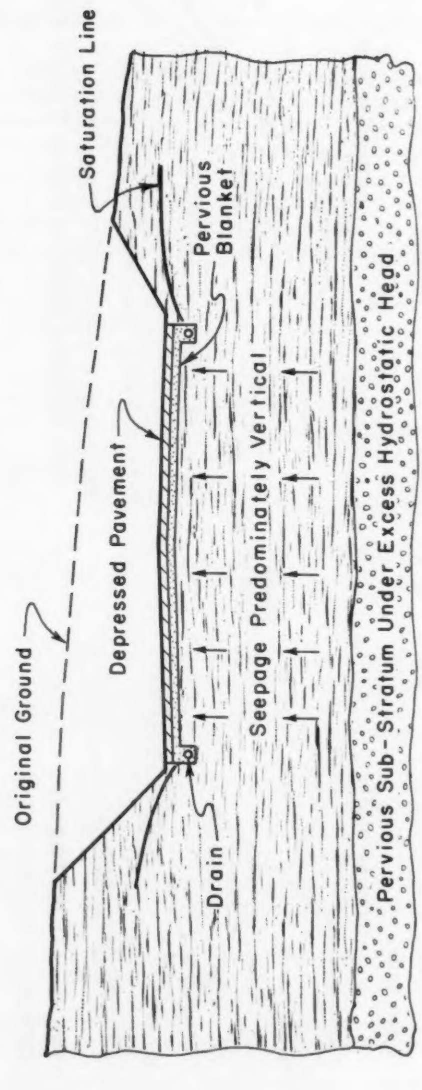


FIG. 2.—EXAMPLE OF ENGINEERING DESIGN UTILIZING RELATIVELY HORIZONTAL DRAINAGE BLANKET FOR REMOVAL OF STEADY SEEPAGE

necessarily prohibit the build-up of head within filters that are required to operate under small gradients.

In general terms, filters often are considered to have adequate permeability if they are "somewhat more pervious" or simply "more pervious" than the protected soil. Such standards (or lack of standards) may not guarantee adequate permeability for removal of seepage. In fact, the required permeability of filters varies substantially with their shape and oreintation. Steeply inclined filters, for example, have relatively high gradients available to hasten the discharge of seepage, whereas nearly flat or horizontal "pervious" bases have relatively minute gradients available.

This paper presents a method for determining the effect of boundary conditions and gradients upon the design requirements of filters subjected to steady seepage. Typical solutions are developed to illustrate a procedure for designing filters and pervious bases that will discharge seepage without excessive build-up of head due to resistance within the water-removing layer.

The condition of "steady seepage" as used herein is not necessarily a year-around condition, but may be of temporary duration. It implies the worst conditions that are likely to prevail for a sufficient length of time to have harmful effects.

PERMEABILITY AND ORIENTATION OF FILTERS CAN BE IMPORTANT

Ordinarily, the permeability of filters and "pervious" bases is not specified, and seldom are hydraulic conditions within such layers analyzed. Under some common usages, the permeability of filters is not particularly critical, and adequate water-removing capacity is easily obtained. Possibly, for this reason, there have been no generally established permeability criteria for filters, pervious bases, and blankets. Casagrande and Shannon[2], however, recognized that the speed with which water can drain from saturated base courses for airfields is directly related to permeability.

The factors influencing the quantity of water flowing through porous media under non-turbulent conditions are expressed by Darcy's law;

$$Q = k\, i\, A \qquad (1)$$

Thus, the seepage quantity, Q, is proportional to the permeability, k, gradient, i, and cross sectional area, A.

If any two of these factors are held constant, the water-removing potential is increased or decreased in direct proportion to changes in the third. Thus, the water-removing capacity of a filter of given cross section and hydraulic gradient, is directly proportional to its permeability. Also, if the permeability and gradient are held constant, the capacity for removing water is directly proportional to the cross sectional area. Recognition of the fact that all three of these factors influence water-removing capacity of filters helps to clarify the basic problem.

Seepage gradients are influenced primarily by hydrostatic head imposed by natural or man-made conditions and boundary conditions established by design. The solutions presented show that the water-removing potential of filters and pervious bases can be increased by providing either greater thickness or greater permeability. The examples are basically "single-layer" filters. If one applies

the principle noted above that water-removing capacity is proportional to permeability one might conclude that any quantity of seepage can be removed simply by providing great enough permeability in the filter. Of course, this may lead to situations where the filter would become so coarse as to allow migration of the fines of the protected layer into or through the filter. When sufficient water-removing capacity cannot safely be assured by means of a single-layer filter some other means must be used, such as a multiple-layered system, or graded filter.

ANALYSIS OF HYDRAULIC CONDITIONS WITHIN FILTERS

General Analysis.—The solutions to be presented show the relative influence of seepage gradients as affected by boundary conditions, and the interrelationship between the permeability ratio (ratio of permeability of filter to permeability of soil) and the physical dimensions of water-removing filter layers.

The influence of boundary conditions and gradients will be shown by analysis of two common, but distinctly different kinds of drainage systems: (a) a steeply inclined filter such as might be placed at the boundary between an "impervious" zone of an earth dam or levee and a "pervious" downstream zone, or against the excavated slope of a "stabilization" trench keying a side-hill fill into firm ground and removing harmful ground water (See Fig. 1); and (b) a flat or horizontal filter or pervious base, such as might be placed beneath a highway or other pavement to intercept rising ground water (See Fig. 2).

The two kinds of design were analyzed by means of flow-nets. Fairly broad ranges of conditions were analyzed and summarized in charts.

The shape of the flow pattern characteristic of any given cross section adjusts to the existing physical conditions, as water will always follow the lines of least resistance. When true conditions at a given location can be approximated by simplified cross sections the flow-net offers a means of predicting paths of flow and distribution of hydrostatic head. Commonly, flow-nets are used in analyzing seepage through dams, levees, and foundations; however, the principle had been applied to diverse situations, including problems of steady-state heat flow, electrostatics, current flow through conductors, pressure distribution in spillways, certain cases in the theory of the torsion of elastic rods, and the flow of viscous liquids.[9,10]

In two-dimensional hydraulic flow, the flow-net is a network of two intersecting families of curves. One family is called the stream lines and represents paths of flow, and the other family the equipotential lines or simply the "potential lines." These networks may be obtained from models or by graphical sketching. Those developed in the present study were obtained by the graphical method, which is readily adapted to the problem under discussion.

One must always recognize that flow-net studies are no more accurate than the degree to which the simplified sections represent true conditions, and true conditions may be extremely difficult to evaluate. Once a set of conditions has been set up, however, there is but one solution,[9] regardless of the method used in finding the solution. Some of the most familiar types of flow-nets are for

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9 "The Flow of Homogeneous Fluids Through Porous Media," by M. Muskat, McGraw-Hill Book Co., Inc., New York, N. Y., 1937, pp. 139-140.

10 "The Flow-net and the Electric Analogy," by E. W. Lane, F. B. Campbell, and W. H. Price, Civil Engineering, October, 1934, p. 510.

homogeneous cross sections, assumed to be of one permeability throughout;[11,12,13] however, some rather complex steady-state seepage problems have been solved[14,15] and approximate nonsteady conditions[16,17] analyzed by this method. Techniques for constructing flow-nets have been capably outlined in engineering publications.[18]

The case under consideration is that of flow from a soil mass of one permeability (the subsoil, or compacted earth) into a porous zone of higher permeability (the filter or pervious blanket). It may be considered a "two-permeability" case, with an indeterminate saturation line which is located simultaneously with construction of the flow-net.

In preparing to construct a flow-net one must establish the physical conditions defining the cross section to be analyzed, including the shape of the porous masses, their average or effective permeabilities, and hydraulic conditions at the edges of the section, or the "boundary conditions." By graphical means (or models) one obtains for a given section the two sets of intersecting lines that have previously been noted—the "flow-lines" and the "potential lines." For a given cross section the resulting pattern has a very definite shape. By constructing a number of nets for varied conditions the relationship may be established between the distance saturation penetrates a "pervious" filter or blanket and the ratio of the permeability of the soil to the permeability of the pervious layer. This relationship is the basis for the typical design charts presented herein.

For each of the two cases analyzed, flow-nets were constructed for a fairly broad range of conditions, and the results summarized on charts. The detailed conditions assumed are shown on the charts and are described in the text.

Sloping Filters.—The first case to be described is not a steeply inclined filter used with construction of the general types illustrated in Fig. 1. Typical flow-nets for this case are shown in Fig. 3 and the solutions have been plotted in Fig. 4 for slopes from $1\frac{1}{2}$:1 to 6:1. A more detailed chart for a $1\frac{1}{2}$:1 slope is given in Fig. 5.

Key dimensions for the section are the height of the saturated discharge face, H, (see Fig. 4) and the depth of penetration, T, of saturation into the filter. This distance is expressed non-dimensionally as the ratio H/T on Figs. 3, 4, and 5.

The slope of the discharge face is designated as S and is expressed as 1 on S. Thus S = 1.5 means a slope of 1 vertically to 1.5 horizontally. The permeability of the soil is designated as k_s and the permeability of the filter k_f. An

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11 "Fundamentals of Soil Mecahnics," by D. W. Taylor, John Wiley & Sons, Inc., N. Y., 1948, p. 157.

12 Discussion by A. Casagrande of ASCE Transactions Paper No. 1919, Vol. 100, 1935, p. 1291.

13 "Methods of Analysis of Flow Problems for Highway Subdrainage," by B. McClelland and L. E. Gregg, Proceedings, Highway Research Board, 1944, pp. 364-376.

14 Discussions of ASCE Transactions Paper No. 2270, Vol. 111, 1946, pp. 236 and 241-244.

15 "Use of Flow-net in Earth Dam and Levee Design," by H. R. Cedergren, Proceedings, Second International Conference on Soil Mechanics and Foundation Engineering, June 21-30, 1948, pp. 293-298.

16 Civil Engineering, August, 1941, p. 499.

17 Discussion by H. R. Cedergren of ASCE Transactions Paper No. 2356, Vol. 113, 1948, pp. 1285-1293.

18 "Seepage Through Dams," by A. Casagrande, Journal, New England Water Works Assn., June, 1937.

impervious boundary is assumed at the elevation of the bottom of the filter. Horizontal and vertical permeabilities are assumed to be equal. A gravel pocket surrounds a longitudinal drainage pipe that permits the free discharge of seepage.

Examination of typical flow-nets for this case (Fig. 3) indicates that seepage in the soil is predominately horizontal. At the boundary between the soil

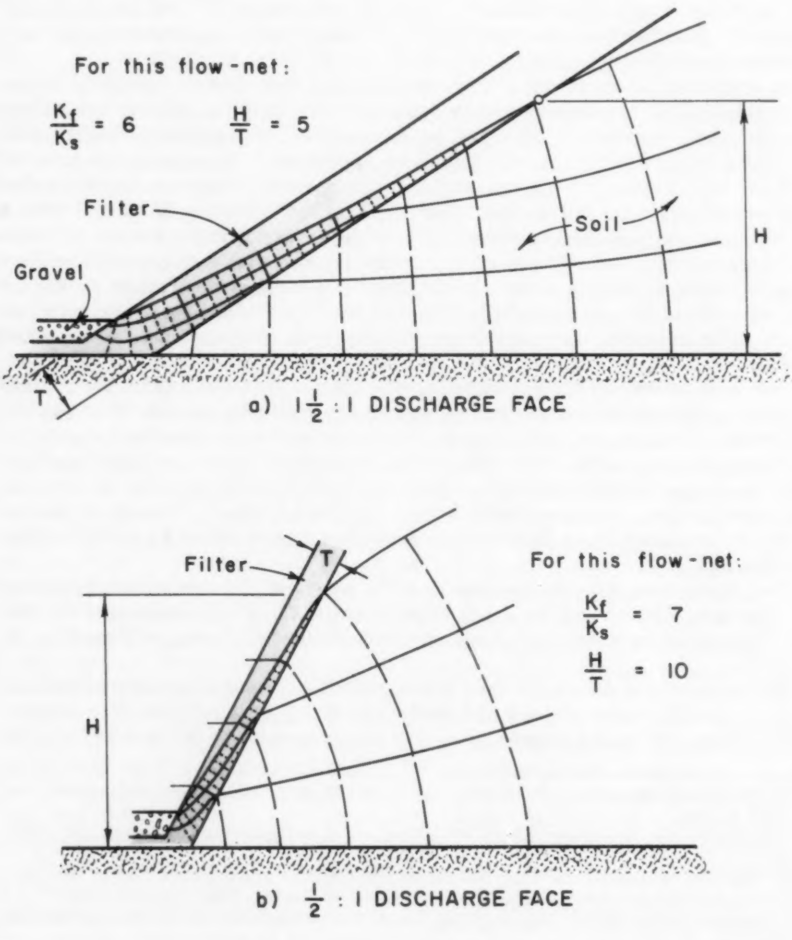


FIG. 3.—TYPICAL FLOW-NETS FOR SEEPAGE INTO SLOPING FILTERS

and the more pervious filter, the flow-lines deflect downward. As seepage accumulates in the filter the thickness of penetration increases to its maximum value (shown as T), which will just contain the zone of saturation. The depth of penetration for a given cross section is dependent upon the ratio of k_f to k_s.

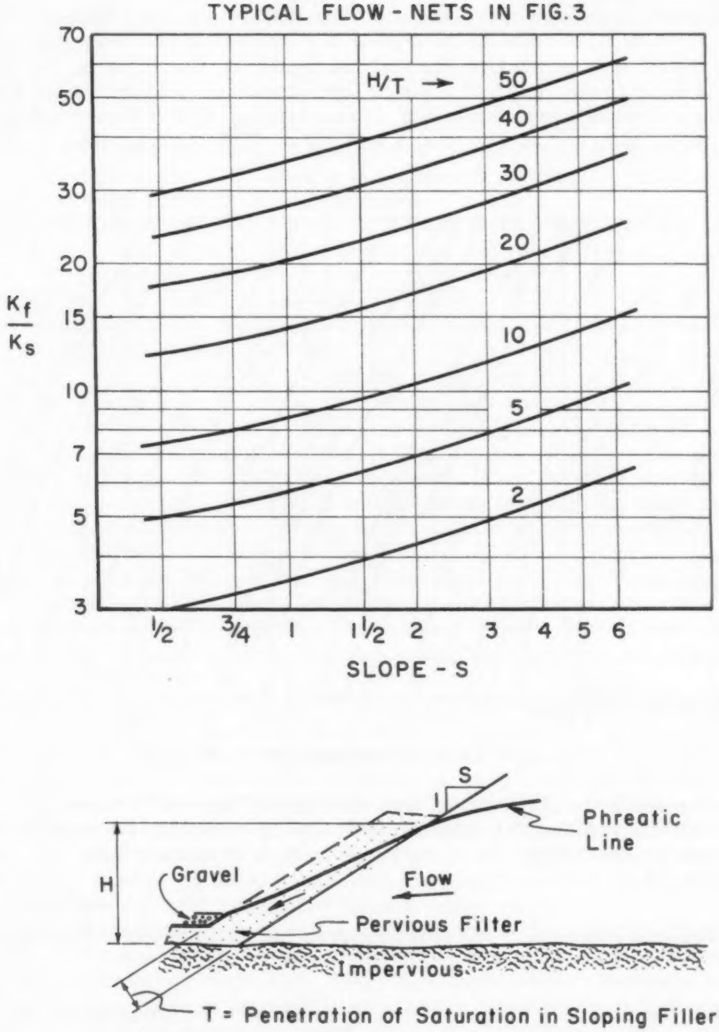
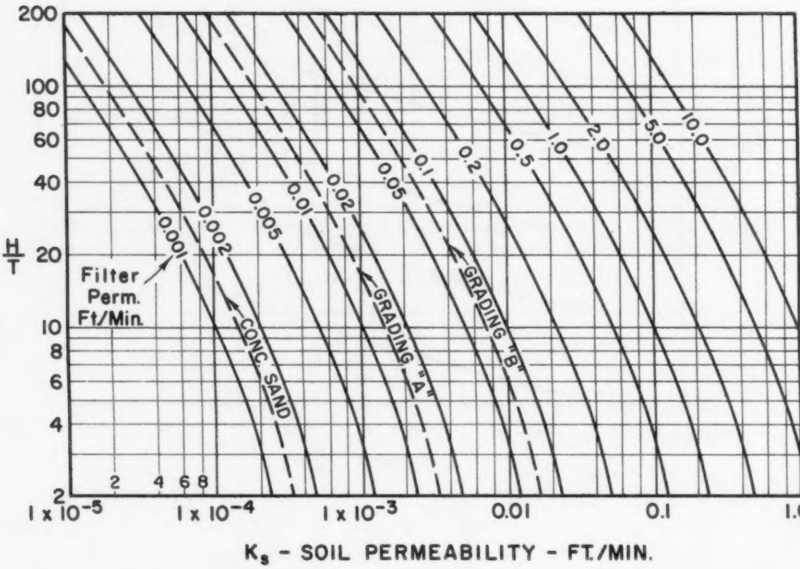


FIG. 4.—FLOW-NET SOLUTION FOR SEEPAGE INTO SLOPING FILTERS ON VARIOUS SLOPES

Graphical checks establish criteria that control the shape of the flow-net and determine the specific ratio k_f/k_s that corresponds to a specific value of T/H. One may select a value of T/H, and construct a balanced flow-net from which the ratio k_f/k_s may be determined or a k_f/k_s ratio may be selected and the depth of penetration determined from the balanced flow-net. In a balanced flow-net the ratio, k_f/k_s is equal to the degree of elongation of the rectangles comprising the net as it passes from the soil into the filter. Numerically, the ratio, k_f/k_s, is equal to the length of a single flow-channel in the filter to its width. Thus, for the conditions shown in Fig. 3(a) a value of H/T of 5 corresponds to a k_f/k_s ratio of 6, and in Fig. 3(b) a H/T ratio of 10 corresponds to a k_f/k_s



Explanation: This chart was developed from H/T values obtained by flow-net solution of steady seepage for boundary conditions shown in diagram in Fig. 4. Typical Flow-nets in Fig. 3.

FIG. 5.—DESIGN CHART FOR PREVIOUS FILTERS OR BLANKETS ON $1\frac{1}{2}$: 1 SLOPE

ratio of 7. As the ratio k_f/k_s becomes greater (and the permeability of the filter greater with respect to the soil) the depth of penetration of saturation into the filter becomes less. This conclusion is evident from scrutiny of the curves in Figs. 4 and 5.

A number of flow-nets similar to the samples in Fig. 3 were constructed, and the results summarized in Fig. 4. The curves in Fig. 4 give the relationship between slope, S, k_f/k_s ratio, and H/T ratio. From Fig. 4 any one of the

three variables may be determined if the other two are known. Thus, if the slope is known, and the permeability of the filter, k_f, and the permeability of the soil, k_s, are known, the ratio H/T may be read from the chart. For the probable height of the saturation face, H, which is estimated from field and soil conditions, the minimum adequate thickness of filter at the bottom of the slope is determined.

To simplify application of the flow-net solutions, detailed charts such as the one reproduced in Fig. 5 may be prepared. Fig. 5 permits direct reading of design ratios (H/T) for given soil permeabilities, k_s, and filter permeabilities, k_f. One chart of the type illustrated in Fig. 5 is required for each slope. Thus, Fig. 5 gives the solution for a $1\frac{1}{2}$:1 slope. It provides the design criteria used in the first of the numerical examples.

Examination of the flow-nets in Fig. 3 for a steeply sloping filter indicates that the total head, H, is, in some degree, available to induce flow within the filter. The significance of this condition on performance will become evident by comparison with the second case.

Horizontal Filters.—The second type of filter installation analyzed by flow-nets is a horizontal "pervious" base for the general type of design illustrated in Fig. 2. Sample flow-nets are presented in Figs. 6 and 7. The pervious blanket is assumed to be horizontal. Water is removed from this layer by a series of shallow, longitudinal drains at a spacing, D. A hydrostatic excess head of 0.2 D is assumed at the top of this aquifer. This head produces an average vertical gradient of 0.4 in the subsoil. The flow-nets give the height of rise of saturation, h, within the filter, for steady seepage excluding capillary rise. The ratio of the distance between drains to the height of rise of saturation due to steady seepage is expressed nondimensionally as D/h. This ratio is related to the permeability ratio (k_f/k_s). Examination of the flow-nets in Figs. 6 and 7 indicates that flow within the subsoil is essentially vertical, while flow in the filter is essentially horizontal. In contrast to the relatively high gradients available in the sloping filter, the horizontal filter has very small gradients available to discharge the incoming seepage.

These small gradients have a retarding effect on seepage discharge that must be compensated by use of filter material of relatively high permeability. This requires the use of relatively coarse and clean material to remove seepage, which in turn means larger voids and greater ease of infiltration. Fortunately, vertical upward flow is the safest from the standpoint of soil migration, since the weight of the soil particles is acting opposite to the direction of flow. An uplift gradient of approximately 1.0 is needed to overcome the downward force exerted by the soil particles, and place the soil in suspension. In the examples, an uplift gradient of 0.4 was assumed.

The relationships derived from these flow-nets are summarized in the chart in Fig. 8. This chart gives the relationship between the permeability ratio (k_f/k_s) and the design ratio, D/h. This chart permits determination of the ratio D/h for a wide range of permeabilities. It permits determination of any one of the variables if the others are known. To facilitate the application of this solution the more detailed plot in Fig. 9 has been prepared. Some computed thicknesses of pervious horizontal bases required for the removal of steady seepage for the assumed boundary conditions are summarized in Table 1.

The practical significance of these solutions will now be illustrated by numerical examples. It might be well to introduce the reminder that the solutions

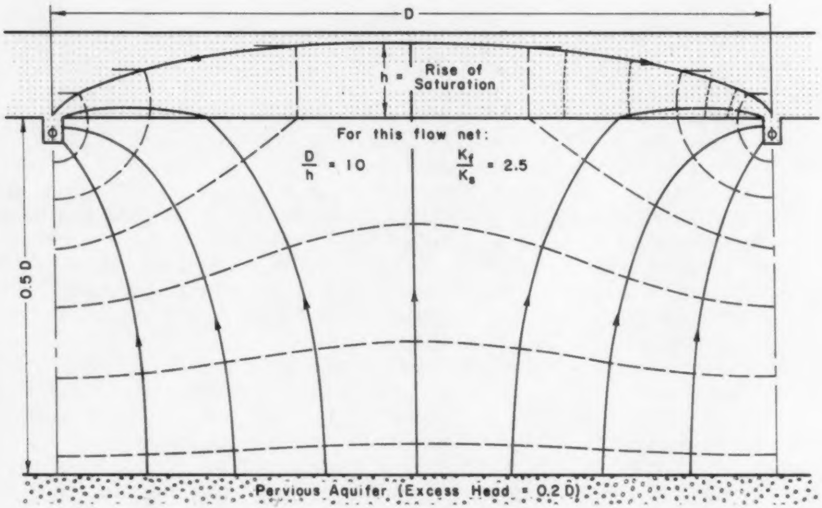


FIG. 6.—TYPICAL FLOW-NET FOR VERTICAL SEEPAGE INTO HORIZONTAL BLANKET

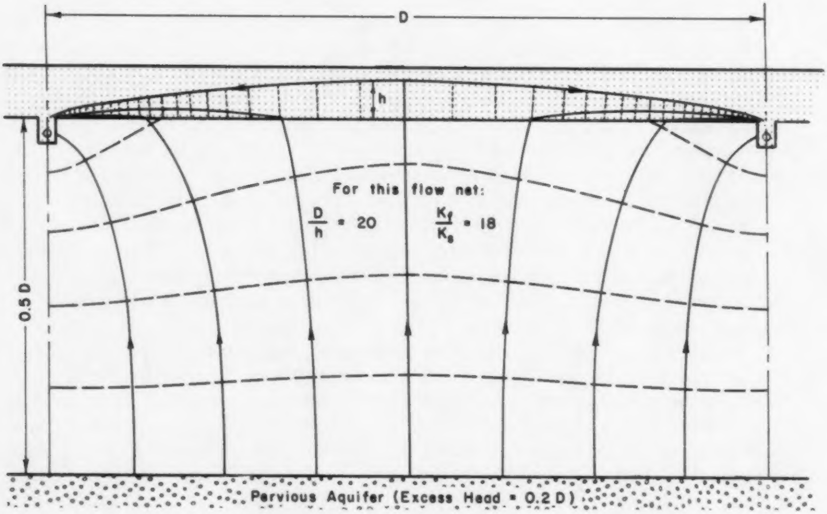


FIG. 7.—TYPICAL FLOW-NET FOR VERTICAL SEEPAGE INTO HORIZONTAL BLANKET

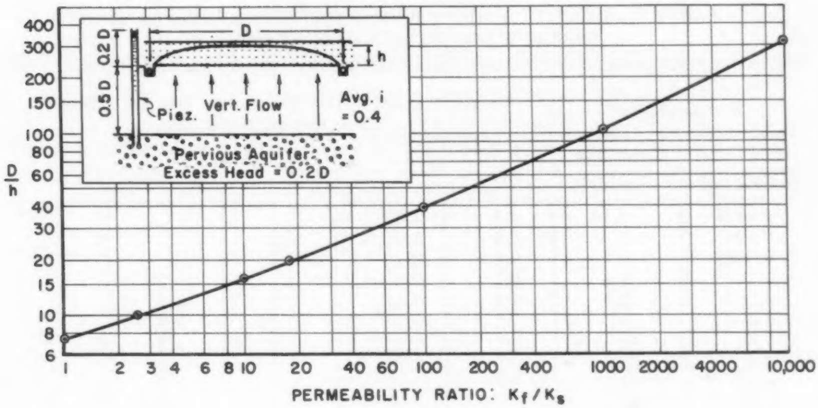


FIG. 8.—FLOW-NET SOLUTION FOR VERTICAL SEEPAGE INTO HORIZONTAL BLANKET

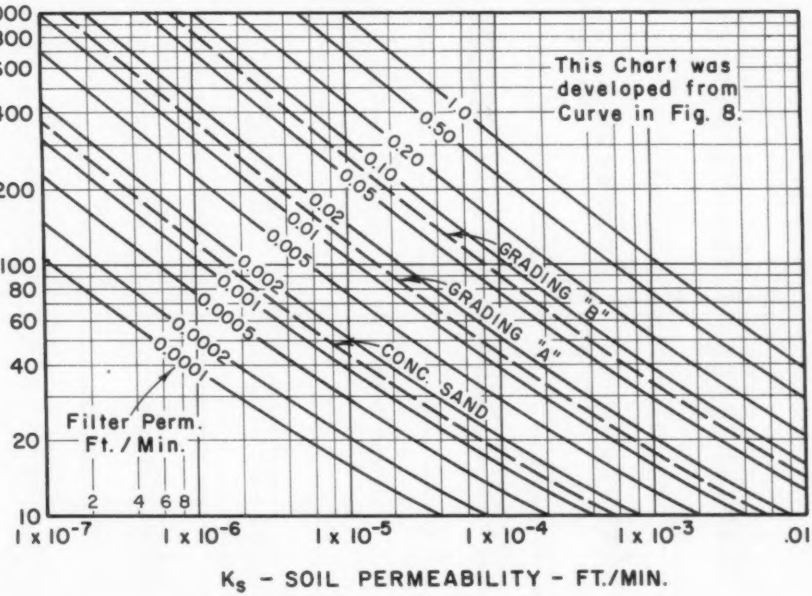


FIG. 9.—DRAIN DESIGN CHART FOR HORIZONTAL BLANKET WITH SHALLOW COLLECTOR DRAINS

TABLE 1.—HEIGHT OF RISE OF SEEPAGE IN PERVIOUS BASE WITH SHALLOW LONGITUDINAL DRAINS[a,b]

Permeability		Height of Rise of Steady Seepage, feet for Effective Soil Perm., ft per min				
ft per min	ft per day	1×10^{-6}	1×10^{-5}	1×10^{-4}	1×10^{-3}	1×10^{-2}
Drains at 20 ft spacing[c]						
0.0001	0.14	0.5	1.3			
0.0002	0.28	0.4	1.0	2.2		
0.0005	0.7	0.3	0.7	1.6		
0.001	1.4	0.2	0.5	1.3		
0.002	2.8	0.1	0.4	1.0	2.2	
0.005	7		0.3	0.7	1.6	
0.01	14		0.2	0.5	1.3	
0.02	28		0.1	0.4	1.0	2.2
0.05	70			0.3	0.7	1.6
0.10	140			0.2	0.5	1.3
0.20	280			0.1	0.4	1.0
0.50	700				0.3	0.7
1.00	1400				0.2	0.5
Drains at 30 ft spacing[c]						
0.0001	0.14	0.8	1.9			
0.0002	0.28	0.6	1.5	3.3		
0.0005	0.7	0.4	1.0	2.4		
0.001	1.4	0.3	0.8	1.9		
0.002	2.8	0.2	0.6	1.5	3.3	
0.005	7	0.1	0.4	1.0	2.4	
0.01	14		0.3	0.8	1.9	
0.02	28		0.2	0.6	1.5	3.3
0.05	70		0.1	0.4	1.0	2.4
0.10	140			0.3	0.8	1.9
0.20	280			0.2	0.6	1.5
0.50	700			0.1	0.4	1.0
1.00	1400				0.3	0.8
Drains at 40 ft spacing[c]						
0.0001	0.14	1.0	2.5			
0.0002	0.28	0.8	2.0	4.4		
0.0005	0.7	0.5	1.4	3.2		
0.001	1.4	0.4	1.0	2.5		
0.002	2.8	0.3	0.8	2.0	4.4	
0.005	7	0.2	0.5	1.4	3.2	
0.01	14	0.1	0.4	1.0	2.5	
0.02	28		0.3	0.8	2.0	4.4
0.05	70		0.2	0.5	1.4	3.2
0.10	140		0.1	0.4	1.0	2.5
0.20	280			0.3	0.8	2.0
0.50	700			0.2	0.5	1.4
1.00	1400			0.1	0.4	1.0

[a] Boundary conditions shown in Fig. 8.
[b] Steady discharge from subgrade under vertical gradient of 0.4.
[c] With pervious base.

TABLE 1.—CONTINUED

Permeability		Height of Rise of Steady Seepage, feet for Effective Soil Perm., ft per min				
ft per min	ft per day	1×10^{-6}	1×10^{-5}	1×10^{-4}	1×10^{-3}	1×10^{-2}
Drains at 50 ft spacing[c]						
0.0001	0.14	1.3	3.1			
0.0002	0.28	1.0	2.5	5.5		
0.0005	0.7	0.7	1.8	4.0		
0.001	1.4	0.5	1.3	3.1		
0.002	2.8	0.4	1.0	2.5	5.5	
0.005	7	0.3	0.7	1.8	4.0	
0.01	14	0.2	0.5	1.3	3.1	
0.02	28		0.4	1.0	2.5	5.5
0.05	70		0.3	0.7	1.8	4.0
0.10	140		0.2	0.5	1.3	3.1
0.20	280			0.4	1.0	2.5
0.50	700			0.3	0.7	1.8
1.00	1400			0.2	0.5	1.3
Drains at 75 ft spacing[c]						
0.0001	0.14	1.9	4.7			
0.0002	0.28	1.4	3.7	8.3		
0.0005	0.70	1.0	2.5	6.0		
0.001	1.4	0.7	1.9	4.7		
0.002	2.8	0.5	1.4	3.7	8.3	
0.005	7	0.3	1.0	2.5	6.0	
0.01	14	0.2	0.7	1.9	4.7	
0.02	28		0.5	1.4	3.7	8.3
0.05	70		0.3	1.0	2.5	6.0
0.10	140		0.2	0.7	1.9	4.7
0.20	280			0.5	1.4	3.7
0.50	700			0.3	1.0	2.5
1.00	1400			0.2	0.7	1.9
Drains at 100 ft spacing[c]						
0.0001	0.14	2.6	6.3			
0.0002	0.28	1.9	4.9	11.2		
0.0005	0.7	1.3	3.4	8.0		
0.001	1.4	1.0	2.6	6.3		
0.002	2.8	0.7	1.9	4.9	11.2	
0.005	7	0.5	1.3	3.4	8.0	
0.01	14	0.3	1.0	2.6	6.3	
0.02	28		0.7	1.9	4.9	11.2
0.05	70		0.5	1.3	3.4	8.0
0.10	140		0.3	1.0	2.6	6.3
0.20	280			0.7	1.9	4.9
0.50	700			0.5	1.3	3.4
1.00	1400			0.3	1.0	2.6

are illustrative of a rational method for designing filters, and that the charts represent specific solutions.

NUMERICAL EXAMPLES

Description of Filter Materials Used in Examples.—In the examples presented, design requirements will be developed for certain assumed physical conditions. Since the water-removing capacity of pervious filters and bases is related to filter permeability, designs will be developed for filters constructed of materials of several permeabilities. To permit a tie-in with practical experience, several specific filter materials of definite permeability will be considered. The assumed permeabilities and gradients are given in Table 2. The values shown are believed reasonably typical for basically rounded, granular filter materials of durable particles. No implication is made that all filter materials of the indicated gradings will have permeabilities of the values shown,

TABLE 2.—PROPERTIES OF FILTER MATERIALS USED IN EXAMPLES

Material	Percentage Passing Sieve Size or No.						Permeability		
							Approx. Range	Used in Example	
	3/8 in.	4	16	30	50	100	(ft per day)	(ft per day)	(ft per min.)
Concrete Sand	100	95-100	45-80		10-30	2-10	0.01-60	2	0.0014
Grading "A"	100	90-100	30-80	15-50	3-10	0-2[a]	20-400	20	0.014
Grading "B"	100	90-100	25-70	10-40	0-6	0-1[a]	100-1000	100	0.07

[a] To assure cleanness, percent passing No. 100 not to exceed one-fifth of the percent passing No. 50.

because the permeability of soils and filter materials depends not only upon gradation, but also upon density, moisture content during placement, method of placement, and densification, and upon the shape, strength, and surface texture of the soil particles. Wide variations in permeability can be attributed to most of these factors and the only sure way of establishing the permeability of a specific material is by test. The values shown in Table 2 are presented only to permit a demonstration of the influence of permeability upon the design of water-removing, protective filters.

Fine concrete aggregate is considered by many engineers to be an excellent filter material. Concrete sand that complies with AASHO specifications can have a wide range of permeability, depending upon the quantity and properties of the fines, as well as other factors. Relatively clean concrete sand can have permeabilities around 50 ft to 60 ft per day (0.035 ft to 0.042 ft per min), but if it contains the maximum permissible amount of fines (10% of -100 mesh) and is well compacted, its permeability can be in the order of only 0.1 to 0.01 ft per day (0.7×10^{-4} ft to 0.7×10^{-5} ft per min). In the examples, concrete

sand is assumed to have a permeability of 2 ft. per day (0.0014 ft per min). Characteristic data for this material, and two progressively cleaner filter materials used in the examples are summarized in Table 2.

Any of the previously described gradings of sound and durable mineral grains, which are placed with reasonable care to avoid segregation, should be satisfactory as a general purpose filter material for protection of many soils under small to moderate seepage gradients without serious danger of infiltration. For high head usages, or installations where infiltration of clogging could have serious consequences, accepted criteria for the prevention of infiltration or clogging should be applied on the basis of the gradation of filter and soil.

Example No. 1 - Sloping Blanket.—A stabilization trench of the type illustrated in Fig. 1(a) is to be constructed in hilly terrain. High ground water imposes a threat to the stability of a highway fill to be constructed at this location. Conditions assumed for the development of the charts in Figs. 4 and 5 are reasonably similar to the conditions at the site. The thickness and permeability of a pervious filter blanket are to be determined from the chart in Fig. 5.

The assumptions are that the height of saturated discharge face, H, is 20 ft, that the slope equals $1\frac{1}{2}$:1 so that S is $1\frac{1}{2}$, and that the soil permeability, k_s, equals 1×10^{-4} ft per min. From Fig. 5 determine required thickness, T, for the given conditions.

(a) Using fine concrete aggregate with a minimum permeability of 0.0014 ft per min (2 ft per day); from Fig. 5, H/T is determined as 16 and required $T = \left(\frac{20}{16}\right) = 1.25$ ft. (b) Using Grading "A", with a minimum k_f of 0.014 ft per min; from Fig. 5, H/T is found to be 140 and required $T = \left(\frac{20}{140}\right) = 0.15$ ft. Under the proposed conditions it is considered that a 2-ft-thick blanket is about as thin as it is practical to construct, hence, fine concrete aggregate, with a minimum permeability of 0.0014 ft per min is satisfactory. A 1-ft blanket of material "A", with a minimum permeability of 0.014 ft per min would have a water-removing capacity substantially greater than that of a 2-ft blanket of fine concrete aggregate of the specified permeability. The thinner blanket would require only half the material needed for the thicker blanket and would provide a much greater discharge capacity. Under conditions where thin blankets are practical, these potentialities are worth considering.

Example No. 2 - Horizontal Blanket.—A freeway through a metropolitan area is to be located in a trench section for several miles. Normal ground water in the area lies several feet above the pavement grade. A pervious sand and gravel substratum gives every indication of sustaining a plentiful supply of ground water. A permanent drainage system for interception of ground water is being considered as a means of keeping excessive water out of the pavement base to prevent uplift, pumping, and general disintegration of the roadway.

Conditions at this location may be reasonably approximated by the solution developed in Figs. 8 and 9.

The assumptions are that the cross section is comparable to Fig. 8; that collector drains are to be at 80-ft intervals, (D = 80 ft); that there will be horizontal top and bottom surfaces of filter (small slope can be neglected); that the blanket thickness is h + 6 in.; and that the soil permeability, k_s, is as in Example 1 equal to 1×10^{-4} ft per min. From Fig. 9 the blanket thickness required for the assumed conditions is determined. (a) Using fine concrete aggregate with minimum k_f equal to 0.0014 ft per min for $k_f = 0.0014$ and $k_s = 1 \times 10^{-4}$ ft per min $D/h = 18$, and $h = D/18 = 80/18 = 4.4$ ft (from Fig. 9). The

required blanket thickness = 4.4 + 0.5 = 4.9 ft. The quantity of filter material required per foot of an 80-ft blanket = 4.9 x 80/27 = 14.5 cu yd. (b) Using Grading "A", with minimum k_f = 0.014 ft per min for k_f = 0.014 ft per min, and $k_s = 1 \times 10^{-4}$ ft per min D/h = 44, and h = D/44 = 80/44 = 1.8 ft, (from Fig. 9). The required blanket thickness = 1.8 + 0.5 = 2.3 ft. The quantity of filter material, "A", required per foot of blanket = 2.3 x 80/27 = 6.8 cu yd. (c) Using grading "B", with minimum permeability = 0.7 ft per min for k_f = 0.07 ft per min, and $k_s = 1 \times 10^{-4}$ ft per min D/h = 90, and h = D/90 = 80/90 = 0.9 ft (from Fig. 9). The required blanket thickness = 0.9 + 0.5 = 1.4 ft. The quantity of filter material, "B", required per foot of blanket = 1.4 x 80/27 = 4.2 cu yd.

The practical design of drainage systems should take into consideration the spacing and depth of collector drains as well as the thickness and quality of the drainage layers. Solutions of the kind presented in Fig. 9 can permit an economic evaluation of these various factors.

CONCLUSIONS

The solutions presented illustrate application of the flow-net to the analysis of permeability requirements of filters and pervious bases for the removal of steady seepage. A complete analysis should also take into consideration the possibility of temporary flooding and rise in saturation caused by capillarity.

The numerical examples and flow-net solutions point up the relatively major influence of boundary conditions and seepage gradients upon the minimum permeability that will assure the rapid removal of steady seepage.

Application of these methods depends upon knowledge of the effective or average permeability of the saturated media from which water is being drained. This determination is the most difficult part of the analysis, and should have the benefit of an experienced evaluation of all available information, including permeability data, ground water observations, and records of performance of projects in the area when such records are available. Every effort must be made to detect and evaluate the presence of porous seams and joints that are easily overlooked but that contribute substantially to the flow of water.

Charts that are presented provide flow-net solutions for two common types of filter design. While the purpose of the charts is to demonstrate a design method, rather than to provide general solutions, they should have some practical value for filter design in conditions comparable to those assumed. For conditions substantially different individual solutions would be needed.

The analysis presented leads to the conclusion that relatively high permeability is needed in filter materials when boundary conditions restrict discharge gradients to small values. In such cases, coarser gradings of filters may sometimes be permitted without danger of infiltration and clogging. In cases where a single layer cannot safely remove the incoming water without danger of infiltration or clogging, other designs should be used, such as multiple-layer or graded filters. In such cases, the layer in contact with the soil should be designed to prevent clogging or infiltration, and another layer should be designed to provide the required water-removing capacity.

In the record numerical example, the design thickness of the filter layer was taken as 6 in. greater than the theoretical penetration of saturation caused by steady seepage. This 6 in. thickness is a small allowance for capillarity and "free board." It will be recognized that capillary rise can vary from nearly zero in clean pea gravel to several feet in compacted graded filter material

containing appreciable fines. Possible entrapment or "looking" of water beneath pavements by capillarity lends support to the desirability of thinking in terms of materials cleaner than are commonly used in pervious bases.

The solutions point to the desirability of a general appraisal of some commonly accepted specifications for filter materials. Classes of materials should be provided that will not only assure protection against infiltration when this protection is needed, but will have sufficient water-removing capacity to meet the needs of individual installations. Because any analysis of this kind can only approximate true conditions, reasonable margins of safety should be provided. Flow-net solutions can furnish a yardstick that is consistent with available knowledge of seepage gradients and soil conditions.

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MECHANICS OF THE TRIAXIAL TEST FOR SOILS

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SYNOPSIS

The triaxial test is analyzed against the background of plastic theory. It is shown that the equilibrium conditions and a failure criterion such as Coulomb's law are insufficient to define a unique failure load; a stress-strain relationship is also needed. The problem is then solved completely for an ideally plastic material. The results are used to discuss the significance of test data for sands and an alternative yield criterion is suggested on the basis of the test data.

INTRODUCTION

It is customary to base both the analysis of the stability of soil masses and the interpretation of soil tests on the Coulomb theory of internal friction. This theory provides what is called, in the terminology of the plasticity theory, is a yield or failure surface and, together with the equilibrium conditions, enables the failure loads to be computed whenever the problem is statically determinate. Other problems that are not intrinsically so can be rendered statically determinate by invoking the hypothesis (1)[2] of A. Haar and Theodore von Kármán, Hon. M. ASCE, asserts that the intermediate principal stress is equal to either the largest or the smallest principal stress. This hypothesis is particularly useful in axially symmetric problems.

This conventional approach suffers from the drawback that the solutions so obtained are not necessarily unique. To assure uniqueness, a stress-strain relationship of a particular type is required (2) and the establishment of such a

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[2] Numerals in parentheses, thus (2), refers to corresponding items in the Bibliography.

relationship, if it exists, is one of the most important of the unsolved problems of soil mechanics.

This paper is concerned with an examination of the so-called triaxial test in the light of a particular stress-strain relationship that would assure stability of the material and uniqueness of failure loads, if its validity could once be established by means of tests. The analysis is used to discuss the significance of tests on sand in which the specimens are caused to fail with axial extension rather than with the more usual axial compression.

The Triaxial Test.—A detailed account of the triaxial test has been presented (3) by A. W. Bishop and D. J. Henkel. The essentials of the device are indicated in Fig. 1. A cylindrical specimen of soil is enclosed within a rubber membrane between stiff end plattens. Hydrostatic pressure is applied by means of an enveloping fluid, that is isolated from the specimen by the rubber membrane, and an extra axial thrust or pull can be exerted on the end plattens. During a test, the relative displacement of the end plattens is observed, and sometimes also the change in volume of the specimen and its change in diameter at a particular height. Testing proceeds by varying the axial load or the lateral pressure or, quite commonly, by imposing relative axial displacements of the plattens and observing the resultant axial loads.

This apparatus now occupies a central position among the devices used to measure the strength of soils. It appears to have developed from a machine designed at the Prussian Waterways Experimental Station for the purpose of studying the consolidation of clays under conditions of negligible side friction (4). In this first apparatus the surrounding liquid was entirely confined, and temperatures and leakage had to be closely controlled to obtain consistent results. Several investigators recognized that the apparatus could be used to measure the ratio of the axial and lateral pressures both prior to and at failure, the first results appearing to be those (5) of T. C. Stanton and F. N. Hveem. Positive control over the lateral pressure was developed independently by L. Rendulic (6) working in Vienna (and later in Berlin, at DEGEBO) and by W. S. Housel (7) at the University of Michigan.

Throughout the literature, it is assumed that, apart from certain end effects, a homogeneous state of stress is produced in the triaxial soil specimen. This point of view appears to have been taken over from the earlier studies of rocks by von Kármán and others and does not appear to have been questioned since that time. Strictly speaking, however, the problem posed by the test situation is statically indeterminate because only the total thrust on the end plattens, not the pressure distribution, is given as a boundary condition. Even allowing circular symmetry and similarity of conditions on every normal cross section, there are still three unknown stresses σ_r, σ_θ, and σ_z (Fig. 2) and only two equilibrium equations:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad \text{(1a)}$$

and

$$\frac{d\sigma_z}{dz} = 0 \qquad \text{(1b)}$$

If σ_θ is equated arbitrarily to σ_r, σ_r is then constant by Eq. 1a and the stress is homogeneous. This step has never been accepted a priori, but it has seemed

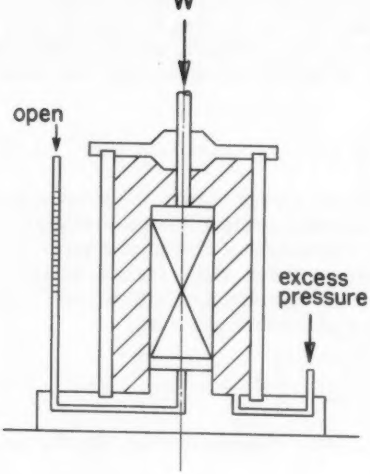

FIG. 1

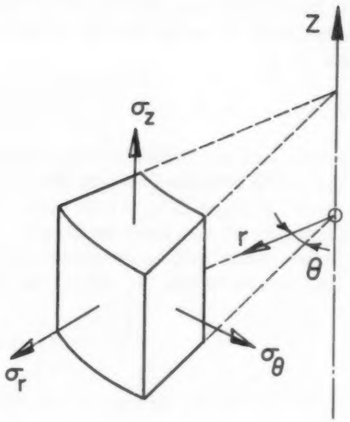

FIG. 2

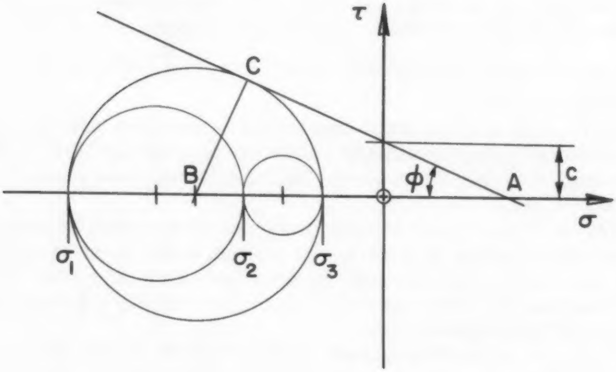

FIG. 3

reasonable in view of its correctness in the elastic range (at sections remote from the ends of the specimen). Moreover, some such assumption has been necessary before information about yield could be extracted from the test.

The Coulomb Yield Criterion.—The weight of evidence from the triaxial test supports the friction criterion of C. A. Coulomb, (8) although minor departures are common. In this criterion the shear stress τ_0 causing slip on any plane is taken as the sum of a constant value, termed the cohesion, and an additional amount that is proportional to the normal pressure acting across the plane. Thus

$$\tau_0 = c - \sigma \tan \phi \qquad (2)$$

in which c is the cohesion, σ denotes the tensile stress across the plane and ϕ is the angle of friction of the material. This relationship follows a straight line in the σ - τ plane (Fig. 3), in which the representation of stress at a point due to O. Mohr (9) can also be drawn. A stress state for which yield is incipient on some plane will be represented by a circle touching the failure line, such as that shown in Fig. 3. From the geometry of the triangle ABC,

$$\frac{\sigma_3 - \sigma_1}{2} = \left[c \cot \phi - \left(\frac{\sigma_1 + \sigma_3}{2}\right)\right] \sin \phi \qquad (3a)$$

in which σ_1 and σ_3 are principal stresses. After rearrangement, Eq. 3a becomes

$$\sigma_1 = \sigma_3 \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2}\right) - 2\, c \tan \left(\frac{\pi}{4} + \frac{\phi}{2}\right) \qquad (3b)$$

It is a property of the Mohr circle that the angle subtended at B (Fig. 3) represents twice the angle of inclination of the plane on which the corresponding stresses act. Thus the critical condition represented by point C in Fig. 3 occurs on a plane the normal of which subtends an angle $\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$ with the normal to the plane on which σ_1 acts.

When the stresses in the triaxial specimen are assumed to be homogeneous, the lateral and axial pressures can be related at once through Eq. 3b. It will be shown subsequently (under the heading "Limiting Equilibrium States") that this solution is not unique.

Stable Plastic Flow.—Lack of uniqueness can be resolved by introducing a stress-strain relationship. It is not known whether a stress-strain relation exists for any particular soil: The purpose here is to make an initial assumption and then to examine the consequences, with a view to obtaining predictions that can be compared with experiments.

We shall assume an isotropic, stable soil that yields at stress combinations that are constant and not a function of the strains or of the stress history. This is the specification for a so-called "ideally plastic" material. A stable material is one from which work cannot be extracted in any loading program, and it follows from this definition (10) that the yield surface of the material is convex when drawn in a space with the stress components as the axes. Also any plastic strain increment vector superimposed on the stress space with corresponding axes matching would lie in the direction of the outwardsdrawn normal to the yield surface. If the yield surface has corners, the vector may lie between adjacent normals.

When the yield surface has a continuously turning tangent, the stresses in a deforming body are uniquely defined (11). If it contains flats, there may be some latitude, although the choice may often be restricted by the requirements of equilibrium (12). For a body with specified surface velocities that are uniform, as in the case of the triaxial specimen deforming between rigid plattens, it may be shown (Appendix I) that the external tractions are unique despite the ambiguity in the internal stressing.

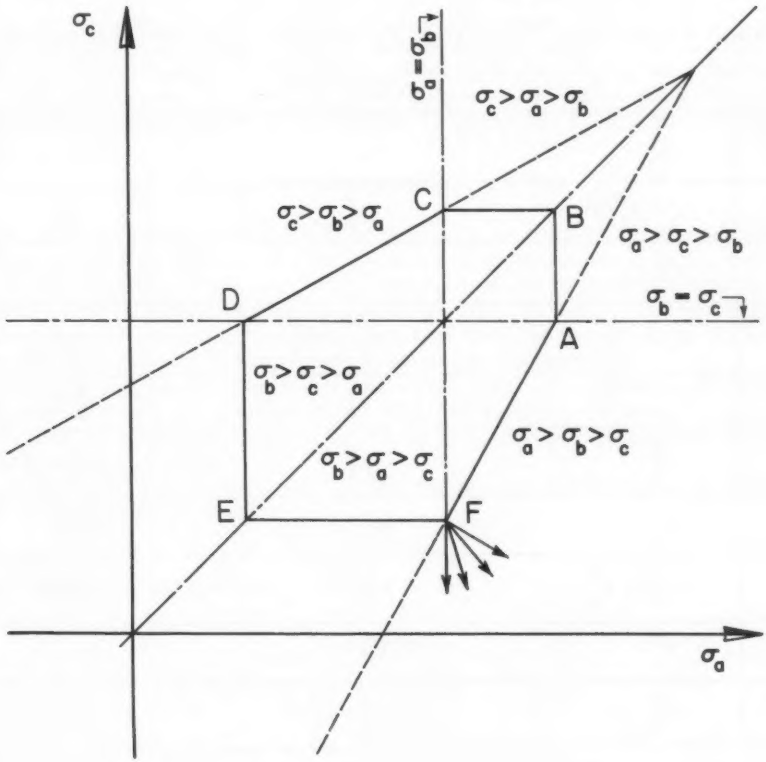

FIG. 4.—COULOMB YIELD CRITERION: INTERSECTION OF THE YIELD SURFACE WITH THE PLANE σ_b = CONSTANT.

The nature of the ideally plastic material which follows the Coulomb yield criterion has been investigated by R. T. Shield (13) and certain results are quoted herewith.

Fig. 4 shows the intersection of the yield surface (Eq. 3b) with a plane in which σ_b is a constant, where σ_b is one of the principal stresses (not necessarily the intermediate principal stress). The regions corresponding to various orderings of σ_a, σ_b, and σ_c are indicated. The strain-rate vectors will lie in the directions of the outward drawn normals to the sides of the cross section

shown provided the components are related in the manner indicated in Table 1. In Table 1, the values λ_1, λ_2, . . . λ_6 are constants and

$$N = \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad \ldots\ldots\ldots\ldots (4)$$

In each line of Table 1, the sum of the strain rates is proportional to $N^2 - 1$ and the sum of the absolute values to $N^2 + 1$; hence

$$\epsilon_a + \epsilon_b + \epsilon_c = \left(\left|\epsilon_a\right| + \left|\epsilon_b\right| + \left|\epsilon_c\right|\right) \sin\phi \geqslant 0 \quad \ldots\ldots (5)$$

in which ϵ is the direct strain rate. At least one of the three components is

TABLE 1

	ϵ_a	:	ϵ_b	:	ϵ_c
A	$\left(\lambda_1^2 + \lambda_6^2\right)N^2$	:	$-\lambda_1^2$	:	$-\lambda_6^2$
AB	$\lambda_1^2N^2$	:	$-\lambda_1^2$	:	0
B	$\lambda_1^2N^2$	:	$-\left(\lambda_1^2 + \lambda_2^2\right)$	:	$\lambda_2^2N^2$
BC	0	:	$-\lambda_2^2$	:	$\lambda_2^2N^2$
C	$-\lambda_3^2$	:	$-\lambda_2^2$	:	$\left(\lambda_2^2 + \lambda_3^2\right)N^2$
CD	$-\lambda_3^2$	:	0	:	$\lambda_3^2N^2$
D	$-\left(\lambda_3^2 + \lambda_4^2\right)$	:	$\lambda_4^2N^2$	:	$\lambda_3^2N^2$
DE	$-\lambda_4^2$	:	$\lambda_4^2N^2$	:	0
E	$-\lambda_4^2$	:	$\left(\lambda_4^2 + \lambda_5^2\right)N^2$	:	$-\lambda_5^2$
EF	0	:	$\lambda_5^2N^2$	:	$-\lambda_5^2$
F	$\lambda_6^2N^2$	:	$\lambda_5^2N^2$	:	$-\left(\lambda_5^2 + \lambda_6^2\right)$
FA	$\lambda_6^2N^2$	:	0	:	$-\lambda_6^2$

positive and at least one negative. If we denote the positive component by ϵ_α and the negative by ϵ_γ, then when the third component ϵ_β is positive

$$\epsilon_\alpha + \epsilon_\beta + \epsilon_\gamma \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) = 0 \quad \ldots\ldots\ldots (6a)$$

and when ϵ_β is negative,

$$\epsilon_\alpha \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) + \epsilon_\beta + \epsilon_\gamma = 0 \quad \text{(6b)}$$

At the apex, the rate of energy dissipation D $\left(= \Sigma\, \sigma_1\, \epsilon_1\right)$ is

$$D = c \cot \phi \left(\epsilon_1 + \epsilon_2 + \epsilon_3\right) \quad \text{(7)}$$

Eq. 7 also applies at all other points on the pyramid because the projection of the stresses in directions normal to the sides is always the same.

Using the ideally plastic material described previously, it is possible to obtain a unique solution for the applied pressures in the triaxial test. This will be done subsequently, but first the lack of uniqueness of the so-called "limiting equilibrium states" will be demonstrated by exhibiting alternative solutions.

LIMITING EQUILIBRIUM STATES

A limiting equilibrium state will be defined as an equilibrium stress distribution in which the soil reaches failure at a sufficient number of points for slip to appear possible. In the case of the triaxial test, limiting equilibrium states can be based on a homogeneous stress distribution, but there are alternatives, based on non-homogeneous distributions, that meet the hypothesis of Haar and von Kármán, and a still wider class of solutions that does not.

For the purposes of illustration a general class of solutions is developed herewith in which the principal shear stresses remain in constant ratio at all points in the specimen, although the stress level may vary from point to point in the radial direction.

Failure with Axial Compression.—Shortening of the specimen will occur when the average axial pressure

$$\bar{q} = -\sigma_z \quad \text{(8a)}$$

exceeds the lateral pressure

$$p = -\left[\sigma_r\right]_{r=a} \quad \text{(8b)}$$

Assuming $\sigma_r > \sigma_z$, Eq. 3b becomes

$$\sigma_z = \sigma_r N^2 - 2\, c\, N \quad \text{(9)}$$

The position of the intermediate principal stress σ_θ can be expressed by the coefficient k where

$$\sigma_\theta = k\, \sigma_r + (1-k)\, \sigma_z \quad \text{(10)}$$

and $1 \geqslant k \geqslant 0$. Substituting Eqs. 9 and 10 into Eq. 1a, integrating and making use of the boundary condition of Eq. 8b

$$\sigma_r = c \cot \phi - (p + c \cot \phi)\left(\frac{r}{a}\right)^{\left(N^2 - 1\right)(1-k)} \quad \text{(11a)}$$

and

$$\sigma_z = c \cot \phi - N^2 (p + c \cot \phi)\left(\frac{r}{a}\right)^{\left(N^2 - 1\right)(1-k)} \quad \text{(11b)}$$

It is confirmed that $\sigma_r > \sigma_z$, because $N^2 > 1$. On integrating σ_z over the normal cross section, the expression for the mean axial pressure $\bar{q}$ becomes

$$\bar{q} = \frac{2\left[N^2 p + (1 + k) N c\right]}{(1 - k) N^2 + 1 + k} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

Setting

$$k = 1 \left(\sigma_\theta = \sigma_r\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

the stress distribution is homogeneous and Eq. 12 reduces to the well known result

$$\bar{q} = p \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2}\right) + 2 c \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad \ldots\ldots\ldots (14a)$$

while on setting $k = 0 \left(\sigma_\theta = \sigma_z\right)$,

$$\bar{q} = (1 + \sin \phi)\, p + c \cos \phi \quad \ldots\ldots\ldots\ldots\ldots (14b)$$

The stress states giving rise to Eqs. 14 both meet the hypothesis of Haar and von Kármán, in that the intermediate principal stress is in each case equal to one of the extreme stresses. In addition, any number of alternative expressions may be obtained from the stress distributions associated with other values of k.

Failure with Axial Extension.—Analogous results can be obtained for the case where the lateral pressure p exceeds the average axial pressure and axial extension occurs during failure. In this case it is assumed that $\sigma_r < \sigma_z$ and Eq. 3b becomes

$$\sigma_r = \sigma_z N^2 - 2 c N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Integrating Eq. 1a after substituting Eqs. 10 and 15,

$$\sigma_r = c \cot \phi - (p + c \cot \phi) \left(\frac{r}{a}\right)^{\left(\frac{1}{N^2} - 1\right)(1 - k)} \quad \ldots\ldots (16a)$$

and

$$\sigma_z = c \cot \phi - \frac{1}{N^2} (p + c \cot \phi) \left(\frac{r}{a}\right)^{\left(\frac{1}{N^2} - 1\right)(1 - k)} \quad \ldots\ldots (16b)$$

Here $\frac{1}{N^2} < 1$, so $\sigma_r < \sigma_z$. On integration of Eq. 16b over the cross section

$$\bar{q} = \frac{2\left[p - (1 + k) N c\right]}{(1 + k) N 2 + 1 - k} \quad \ldots\ldots\ldots\ldots\ldots (17)$$

When $k = 1 \left(\sigma_\theta = \sigma_r\right)$, Eq. 17 gives the usual formula for the homogeneous stress distribution:

$$\bar{q} = p \tan^2 \left(\frac{\pi}{4} - \frac{\phi}{2}\right) - 2 c \tan \left(\frac{\pi}{4} - \frac{\phi}{2}\right) \quad \ldots\ldots\ldots (18)$$

while on setting k = 0 $(\sigma_\theta = \sigma_z)$,

$$\bar{q} = (1 - \sin\phi)\ p - c \cos\phi \quad \ldots\ldots\ldots\ldots\ (19)$$

Again Eqs. 18 and 19 satisfy the hypothesis of Haar and von Kármán, intermediate formulas being obtained by using other values of k.

The ranges of possible values of $\bar{q}$ represented by Eqs. 12 and 17 are illustrated in Fig. 5 for the particular case c = 1 (unit stress) and ϕ = 20°.

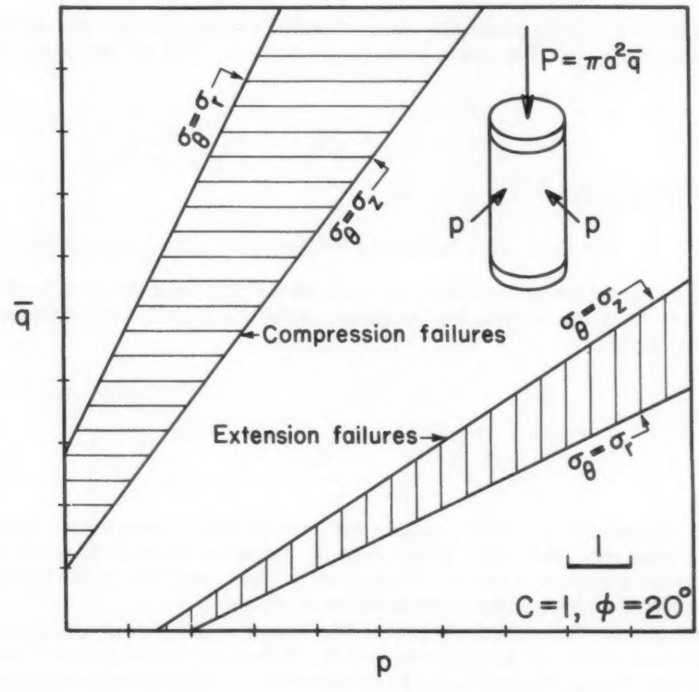

FIG. 5.—VALUES OF LATERAL PRESSURE p AND MEAN AXIAL PRESSURE $\bar{q}$ FOR VARIOUS LIMITING EQUILIBRIUM SOLUTIONS. THE CONVENTIONAL SOLUTION IS REPRESENTED BY THE LINES MARKED $\sigma_\theta = \sigma_r$.

In the case of axial compression, the usual formula (Eq. 14a), represented by the upper boundary of the shaded zone, gives the highest possible values of $\bar{q}$. Loading paths that start at a hydrostatic pressure state ($\bar{q}$ = p) must cross a zone in which, according to limiting equilibrium theory, failure is possible with a non-uniform stress distribution.

In the foregoing illustration of alternative stress states, it is assumed that the principal shear stresses are in constant ratio, although there is no obvious reason why this should be so. If the assumption is abandoned and k made a function of the radius, many other stress distributions, all satisfying the yield condition and the boundary conditions, become possible.

A choice between all these solutions can be made by introducing a stress-strain relationship such as that described in the Introduction. The particular stress-strain relation that has been selected narrows the choice to the point where the external tractions become defined uniquely and a rational comparison with test data becomes possible. With the aid of this relationship, several possible velocity fields, associated with the homogeneous stress state, will be found.

PLASTIC SOLUTION FOR PLANE STRAIN

Suppose a single plane slip band of width t forms across the width of the specimen (Fig. 6). For plane strain $\epsilon_2 = 0$ and ϵ_1 and ϵ_3 are given by

$$\frac{1}{2}(\epsilon_x + \epsilon_y) \pm \frac{1}{2}\left\{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}{}^2\right\}^{1/2}$$

Substituting in either Eq. 6a or 6b,

$$\epsilon_y = \gamma_{xy} \tan \phi \qquad (20)$$

Thus the direction of motion of one side of the slip band relative to the other subtends an angle ϕ with the direction of the band (14). The corresponding velocity field is

$$u_x = \frac{y\, u_0 \tan \phi}{t} \qquad (21a)$$

and

$$u_y = \frac{y\, u_0}{t} \qquad (21b)$$

in the slip band ($t \geqslant y \geqslant 0$). Both in the case of axial compression (Fig. 6 (a)) and of axial extension (Fig. 6 (b)), velocity fields have now been found that are associated with Eqs. 9 and 15, respectively. The remainder of the specimen is not stressed above yield and the solution is complete.

Extent of the Deformable Region.—The extent of the deformable region can now be found by using the theorem of J. W. Bishop, A. P. Green and R. Hill (15) that states that any region shown to be necessarily rigid for a particular stress field (by arguments based solely on the geometrical and other properties of that field) must be rigid in all complete solutions. The slip lines form the characteristics both for the stress (16) and the velocity equations. They serve to isolate regions at the ends of the specimens (as indicated in Fig. 7) which are necessarily rigid because the velocity will be continuous across the surface of the plattens if there is any friction whatever; otherwise σ_z would not remain a principal stress. Fig. 7 can represent any diametral cross section, so the necessarily rigid zones are conical.

Dilatation.—The total dilatation can be computed from the strains (Eq. 20) or directly from energy considerations. By adopting the latter method, the dilatation rate can be shown to be independent of the precise nature of the velocity field. It is most convenient to consider separately the action of the hydrostatic pressure p and the excess axial load Q. Equating the work done by the external

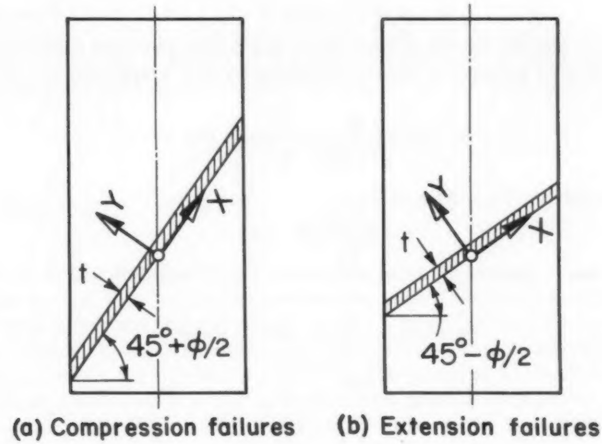

FIG. 6.—POSSIBLE SLIP PLANES.

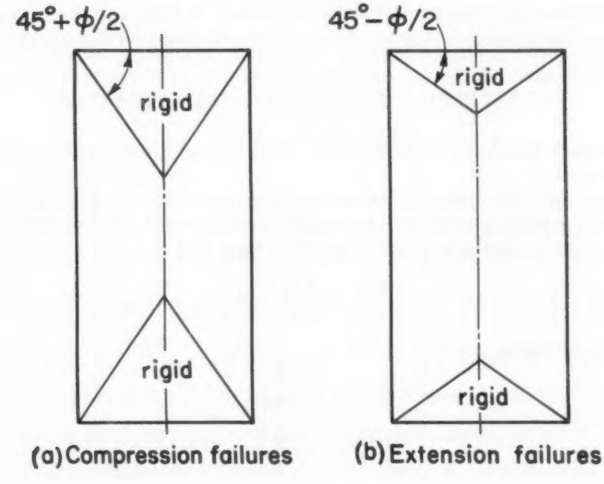

FIG. 7.—EXTENT OF THE DEFORMABLE REGION.

forces during unit axial compression to the internal energy dissipation computed from Eq. 7

$$Q - p\,\Delta = c\,\Delta \cot\phi \quad \text{. . . . . (22)}$$

in which Δ is the dilatation of the entire specimen per unit axial compression.

In the case of failure by axial compression, Eq. 9 applies, and it can be re-written as

$$p = \frac{Q}{\pi a^2 \left(N^2 - 1\right)} - c \cot\phi \quad \text{. . . . . (23)}$$

Hence, comparing Eqs. 22 and 23,

$$\Delta = \pi a^2 \left(N^2 - 1\right) \quad \text{. (24)}$$

In the case of failure by axial extension, Eq. 15 applies, and it can be rewritten as

$$p = \frac{Q}{\pi a^2 \left(\frac{1}{N^2} - 1\right)} - c \cot\phi \quad \text{. . . . . (25)}$$

so in this case

$$\Delta = \pi a^2 \left(\frac{1}{N^2} - 1\right) \quad \text{. . . . . (26)}$$

The foregoing plane-strain solution establishes a possible velocity field for the homogeneous stress distribution $\left(\text{obtained by setting } \sigma_\theta - \sigma_r\right)$but, although it establishes unique external pressure values, it does not ensure uniqueness of the stress distribution, due to the presence of flats on the yield surface.

PLASTIC SOLUTIONS FOR AXIAL SYMMETRY

The analysis given herewith follows closely that given by Shield (17) for the case of a metal ($\phi = 0$).

Assuming that the material remains isotropic, then the principal directions of strain will coincide with the principal directions of stress; hence $\gamma_{rz} = 0$, or, denoting the radial and axial velocities by u and w respectively,

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = 0 \quad \text{. . . . . (27)}$$

Other strain components are

$$\epsilon_r = \frac{\partial u}{\partial r} \quad \text{. . . . . (28a)}$$

$$\epsilon_\theta = \frac{u}{r} \quad \text{. . . . . (28b)}$$

and

$$\epsilon_z = \frac{\partial w}{\partial z} \quad \text{. . . . . (28c)}$$

Now consider possible stress states, Fig. 4, for definiteness setting $\sigma_a = \sigma_r$, $\sigma_b = \sigma_\theta$, and $\sigma_c = \sigma_z$.

Failure with Axial Compression.—In the case of failure by axial compression, $\sigma_r > \sigma_z$ and, when σ_θ is the intermediate principal stress, only state points

on side AF, Fig. 4, need be considered. On side AF, except at A and at F, $\epsilon_\theta = 0$ by Table 1, then $\epsilon_r = 0$ by Eqs. 28 and $\epsilon_z = 0$ by Eqs. 6 hence no solution is possible. Possible solutions are thus immediately restricted to the state points A and F.

At F the term ϵ_θ is positive from Table 1, so Eq. 6a applies. Recalling Eq. 27 and writing Eq. 6a in terms of the velocity components by means of Eq. 28, two simultaneous partial differential equations for the velccities are obtained:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) = 0 \quad \ldots\ldots\ldots (29a)$$

and

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (29b)$$

Shield has solved Eqs. 29 for the particular case of N=1 and solutions for $N \neq 1$ can be found along exactly the same lines. A first deforming zone is shown in Fig. 8 (a), which illustrates one half of the cross section. Continuity of velocity is assumed across 0B so that u = w = 0 on 0B except at O. Neither Eqs. 29 nor the boundary conditions involve a fundamental length, so u and w will be functions of ψ only (see Fig. 8 (a)), and Eqs. 29 reduce to

$$- u' \sin \psi + u \sec \psi + w \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \cos \psi = 0 \ldots (30a)$$

and

$$u' \cos \psi - w' \sin \psi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (30b)$$

where the primes denote differentiation with respect to ψ. The dependent variables can be separated by the substitution x - tan ψ, leading to the solution

$$u = \frac{2}{\pi} \sqrt{\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \tan^2 \psi} \quad \ldots\ldots\ldots (31a)$$

and

$$w = \frac{2}{\pi} \tan^{-1} \sqrt{\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\cot^2 \psi - 1} \quad \ldots\ldots\ldots (31b)$$

where the inverse tangent lies between 0 and π. The strain rates are in the ratio

$$\epsilon_r : \epsilon_\theta : \epsilon_z = \tan^2 \psi : \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \tan^2 \psi : -1 \quad \ldots\ldots (32)$$

so it is confirmed that ϵ_r and ϵ_θ are positive when ϵ_z is negative. The angle ψ varies from zero to $\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$ and the resulting velocity vectors cover the entire range of permissible directions at the point F, as indicated in Fig. 4. The velocity field can be associated with one and only one stress point, so that the stresses are uniquely defined for the velocity field (Eqs. 31) despite the presence of flats on the yield surface. As a consequence it is not necessary to investigate state point A as a possibility. Were this to be done, it would be found that the strains have incorrect signs.

The total dilatation rate has been found previously by energy considerations. It can also be found by integrating the sum of the strain rates over the deforming region.

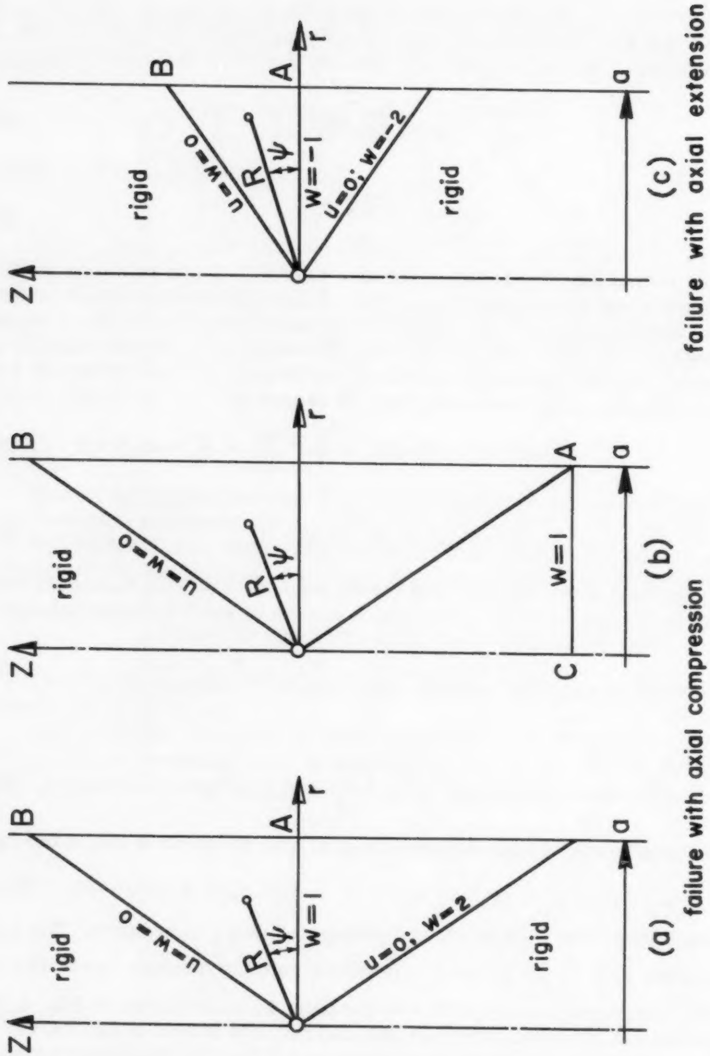

FIG. 8.—REGIONS OF DEFORMATION WITH AXIAL SYMMETRY.

The nature of the deformation (Eqs. 31) is indicated in Fig. 9 (a), which shows the deformed shape of an initially square grid obtained by assuming that the initial velocities are maintained for a finite deformation.

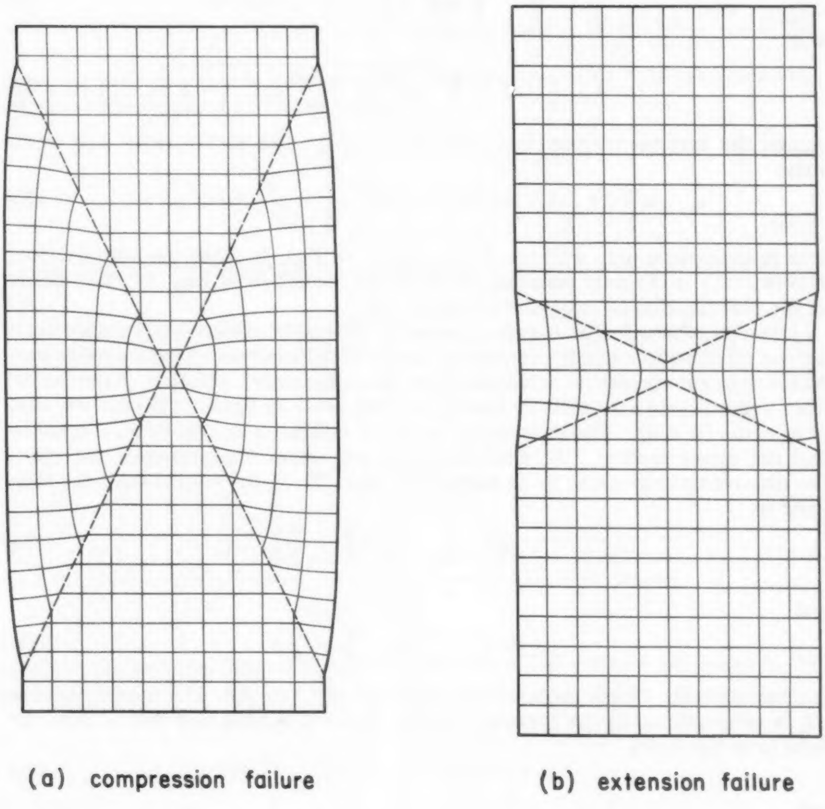

FIG. 9.—DISTORTION OF AN INITIALLY SQUARE GRID FOR DEFORMATION WITH AXIAL SYMMETRY.

Failure with Axial Extension.—In this case $\sigma_r < \sigma_z$ and failure states lie on side CD on Fig. 4. Except at C and D, $\epsilon_\theta = 0$ by Table 1, and no solution is possible. At C, ϵ_θ is negative, so Eq. 6b applies, which with Eq. 27 gives

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) = 0 \quad \ldots\ldots\ldots \quad (33a)$$

and

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (33b)$$

Adopting the deforming region indicated in Fig. 8 (c) and proceeding exactly as in the axial compression case, the velocities are found to be

$$u = -\frac{2}{\pi} \sqrt{\tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) - \tan^2\psi} \quad \ldots\ldots\ (34a)$$

and

$$w = -\frac{2}{\pi} \tan^{-1} \sqrt{\tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right)\cot^2\psi - 1} \quad \ldots\ldots\ (34b)$$

Again, the inverse tangent lies between 0 and π. The strain rates are in the ratio

$$\epsilon_r : \epsilon_\theta : \epsilon_z = -\tan^2\psi : -\tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) + \tan^2\psi : 1 \quad \ldots\ (35)$$

that is consistent only with the state point A in Fig. 4. Thus the homogeneous stress state is the only possible one with the velocities of Eqs. 34. The nature of the deformation is indicated in Fig. 9 (b).

Other Solutions.—The foregoing solution serves to identify the homogeneous stress state with an axially symmetric mode of deformation. That the mode considered is not unique is established by the alternative solution given below. This second field also follows closely a field used by Shield (17) for the case of a metal ($\phi = 0$). The deforming region is indicated in Fig. 8 (b), that shows half the cross section. We shall consider only axial compression, for which the differential equations to be solved are Eqs. 29. In the region OAB the field used is

$$u = \frac{r}{2a} \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad \ldots\ldots\ldots\ (36a)$$

and

$$w = -\frac{z}{a} \tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \quad \ldots\ldots\ldots\ (36b)$$

that satisfies Eq. 29 and the boundary condition $w = 1$ on AB. The velocity across OA is proportional to the distance R from O, so it is assumed that in AOC the field is of the form

$$u = R\,F(\psi) \quad \ldots\ldots\ldots\ (37a)$$

and

$$w = R\,G(\psi) \quad \ldots\ldots\ldots\ (37b)$$

where F and G are functions of ψ only. The condition that the normal velocity across OA be continuous (OA is a slip direction for the stress field being considered) requires

$$F \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) + G = \frac{\left[1 + \frac{1}{2}\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) + 1\right]}{a\sqrt{\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) + 1}} \quad \ldots\ldots\ldots\ (38)$$

on OA and F = G = O on OC. Substitution of Eqs. 36 into Eqs. 29 gives

$$F\,(\cos\Psi + \sec\Psi) - F'\sin\Psi + G\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\sin\Psi$$

$$+ G'\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\cos\Psi = 0 \quad \ldots\ldots\ldots\ldots \quad (39a)$$

and

$$F\sin\Psi + F'\cos\Psi + G\cos\Psi - G'\sin\Psi = 0 \quad \ldots \quad (39b)$$

where the primes indicate differentiation with respect to Ψ. The variables are separated after making the substitutions

$$F = A\,(\Psi)\cos\Psi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (40a)$$

and

$$G = B\,(\Psi)\sin\Psi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (40b)$$

and the solution satisfying the boundary conditions on F and G is found to be

$$u = R\,F = \frac{R\left[\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\cos\Psi\,\tan^{-1}\sqrt{\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\cot^2\Psi - 1} - \sin\Psi\sqrt{\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \tan^2\Psi}\right]}{2\pi a\,\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)} \quad \ldots \quad (41a)$$

and

$$w = R\,G = \frac{R\left[\cos\Psi\sqrt{\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \tan^2\Psi} - \sin\Psi\,\tan^{-1}\sqrt{\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\cot^2\Psi - 1}\right]}{\pi a\,\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)} \quad \ldots\ldots \quad (41b)$$

in which the inverse tangent lies between 0 and π. The strain rates are in the ratio

$$\epsilon_r : \epsilon_\theta : \epsilon_z = 1 + \frac{\gamma\sqrt{1-\gamma^2}}{\cos^{-1}\gamma} : 1 - \frac{\gamma\sqrt{1-\gamma^2}}{\cos^{-1}\gamma} : -2\tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) \quad \ldots \quad (42)$$

in which

$$\gamma = \frac{\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)}{\tan\Psi} \quad \ldots\ldots\ldots\ldots\ldots \quad (43)$$

This is consistent with point F on the yield surface, Fig. 4.

All the solutions presented are associated with the homogeneous stress distribution, the axially symmetric modes being associated with that distribution and no other. While it cannot be proved that other stress distributions are impossible, in view of the flats on the yield surface, they appear highly unlikely. Moreover, their identification, if they exist, would serve no useful purpose because the present solutions give unique values for the external pressure ratio and the dilatation and serve to show that both plane strain and axially symmetric deformation modes do, in fact, exist for an ideally plastic material that fails according to the Coulomb yield criterion.

The solutions described herewith also apply to hollow cylindrical specimens, if the inside and outside radial pressures are equal.

EXPERIMENTAL EVIDENCE

The foregoing analysis enables the triaxial test to be used to check the validity of the hypothesis of Haar and von Kármán by a comparison of the results of extension and compression tests. The complete plastic solutions are of value

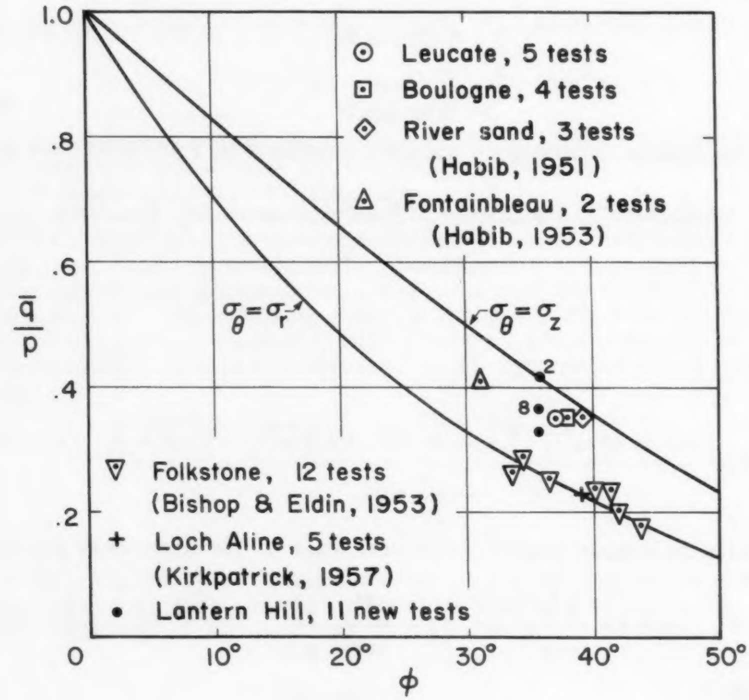

FIG. 10.—TRIAXIAL EXTENSION TESTS ON SAND. RATIO OF AXIAL TO LATERAL PRESSURE VS. ANGLE OF FRICTION COMPUTED ASSUMING $\sigma_\theta = \sigma_r$ IN COMPRESSION TESTS.

in that greater significance is now placed upon the Haar and von Kármán hypothesis, since its validity becomes a necessary (but not sufficient) condition for soil to behave as an ideally plastic material that obeys Coulomb's criterion of failure.

Several series of tests on sands under extension have been described in the literature, and the results are summarized in Fig. 10. The theoretical lines in Fig. 10 were computed by means of Eqs. 16 and represent the extremes of $\bar{q}/p$ theoretically possible while σ_θ remains the intermediate principal stress and

the ratio of the principal shear stresses is constant. The test results are plotted under the assumption that $\sigma_\theta = \sigma_r$ in the compression test. They lie in the expected zone, but there is a considerable discrepancy between the results obtained by M. P. Habib (18) (19) in 1951 and 1953 and those obtained by Bishop and A. K. G. Eldin (20) and by W. M. Kirkpatrick (21) in 1953 and 1957, respectively. A series of tests was undertaken in an attempt to obtain more information, and these new results are also indicated in Fig. 10.

TABLE 2.—RESULTS OF TRIAXIAL TESTS ON CRUSHED QUARTZ (LANTERN HILL)
TABLE 2

Test[a]	Initial Porosity, in %	Lateral Pressure p, in kg per sq cm	Mean Axial Pressure $\bar{q}$, in kg per cm	Dilatation Rate in cc per cm
A1	49	0.6	2.4	4.9
A2	49	0.7	2.7	3.7
A3	51	1.0	3.7	2.6
A4	47	1.4	5.4	2.7
A5	51	1.6	6.0	2.0
A6	-	2.1	8.4	-
A7	49	2.6	10.6	2.2
A8	51	2.6	9.6	1.2
A9	49	2.6	9.8	1.6
A10	50	2.8	10.7	2.3
A11	49	2.8	10.5	-
A12	50	3.0	11.5	1.8
B1	51	0.6	2.4	5.3
B2	50	1.2	4.4	4.7
B3	49	1.8	7.0	-
B4	50	2.4	8.4	1.8
C1	49	1.0	0.3	-1.4
C2	50	2.0	0.6	-
C3	51	3.0	1.2	0
C4	50	4.0	1.3	0
C5	48	5.0	2.1	0
D1	51	1.6	0.6	-0.3
D2	50	3.2	1.2	0
D3	50	3.7	1.2	-0.1
D4	50	4.7	1.6	0
D5	51	5.6	2.1	-
D6	52	6.4	2.4	+0.3

[a] Compression tests: Series A - failure by increasing axial pressure; Series B - failure by decreasing lateral pressure. Extension tests: Series C - failure by decreasing axial pressure; Series D - failure by decreasing lateral pressure.

The new series was run on a crushed quartz from Lantern Hill, Mystic, Conn. The quartz was almost pure, being of glass-making quality. The fraction used in the tests was that retained on a No. 50 sieve (opening 297 μ) and passing a No. 45 sieve (opening 350 μ). The sand was placed dry by tamping in three layers, and was then saturated with water before testing. The mean relative density achieved by this process was 70%.

The detailed results are given in Table 2 and in Fig. 11. Two methods of reaching the failure state were used -- varying the axial load and reducing the

lateral pressure. In each case failure was deemed to have occurred when the modulus was reached to a 10 kg per sq cm in terms of whichever pressure was being varied. The direction of approach to the failure point is indicated in Fig. 11 by a short line running out from each point. A mean line has been drawn through the compression test results, and the angle of friction computed from this line (35.6°) was used to compute the two theoretical lines shown for the extension test using Eqs. 18 and 19. The experimental results for the extension tests fall in between the theoretical lines, the tests with failure by reducing the axial pressure showing some scatter and those with failure by reducing the lateral pressure very little. Typical points are also plotted in Fig. 10 where they fall close to those established by Habib.

Rates of volume increase per unit axial compression are also given in Table 2; these are the average values at failure. It should be noted that in series C and D the specimens were extending, so negative values of the dilatation per unit axial compression represent an expansion of the specimen. Theoretical values of dilatation based on Eq. 24 for compression and Eq. 26 for extension are 28 cc per cm and -7.4 cc per cm, respectively. Thus it is seen that the actual dilatations were much smaller than predicted using an ideally plastic model. No allowances have been made for the elastic compression of the specimens, that would probably vary according to the intensity of the hydrostatic pressure, and would, in all cases, tend to decrease the dilatation.

The specimens deformed either in thin slip bands, as indicated in Fig. 6, or by a general axially symmetric deformation, as in Fig. 9. In the case of failure with axial compression, it was usually easy to decide which method of failure had occurred by examining the specimen at the conclusion of a test. In axial extension tests it was much more difficult to decide, a typical specimen being that shown in Fig. 12. In the case illustrated an axially symmetric neck has formed but there is also some evidence of a shear plane running from top left to bottom right of the sheared zone. In all the extension tests shear was confined to a relatively small zone similar in size to that illustrated in Fig. 9 (b). In compression, however, shearing frequently extended the entire length of the specimen between the plattens. This result is as expected: Changes in geometry tend to reduce the critical cross sections in extension, leading to increased stresses on planes that have slipped; while in compression the cross section will tend to be increased and slip may well be transferred to a smaller, and hence weaker, plane.

The present series of tests tends to support Habib's conclusion that the effective angle of friction in extension tests may sometimes differ from that in compression. Independent tests by Bishop and Eldin and by Kirkpatrick show that the angle is identical for some sands. A considerably greater volume of experimental evidence is, of course, needed before any strong assertions can be made. On the evidence to date it does appear, however, that the mechanical behavior of sands may vary in some fundamental manner, governed by such factors as grading, particle shape, porosity, etc. It will be necessary to make many investigations before a clear picture emerges.

The foregoing interpretation is based on the assumption that a yield criterion of the Coulomb type applies in all cases. The observed differences in the effective angle of friction for extension and compression tests can then be explained only by postulating the development of non-homogeneous stress states. An alternative and more attractive approach is to use the extension tests to obtain new information on the nature of the yield criterion. In order to do this it

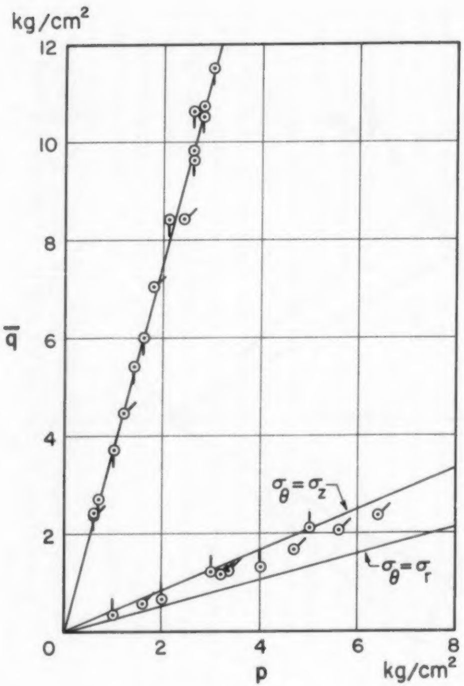

FIG. 11.—TRIAXIAL TESTS ON CRUSHED QUARTZ SAND (LANTERN HILL)

FIG. 12.—APPEARANCE OF A SPECIMEN AFTER AN EXTENSION FAILURE, SHOWING THE LIMITED EXTENT OF THE DEFORMED ZONE (SEE ALSO FIG. 9(b)

must, of course, be assumed that the stress state remains homogeneous under all circumstances, otherwise it would not be possible to deduce the stresses from the observed pressures.

When homogeneity of stress is assumed, it is then possible to plot experimental data in a space with the principal stresses as axes. Furthermore, since

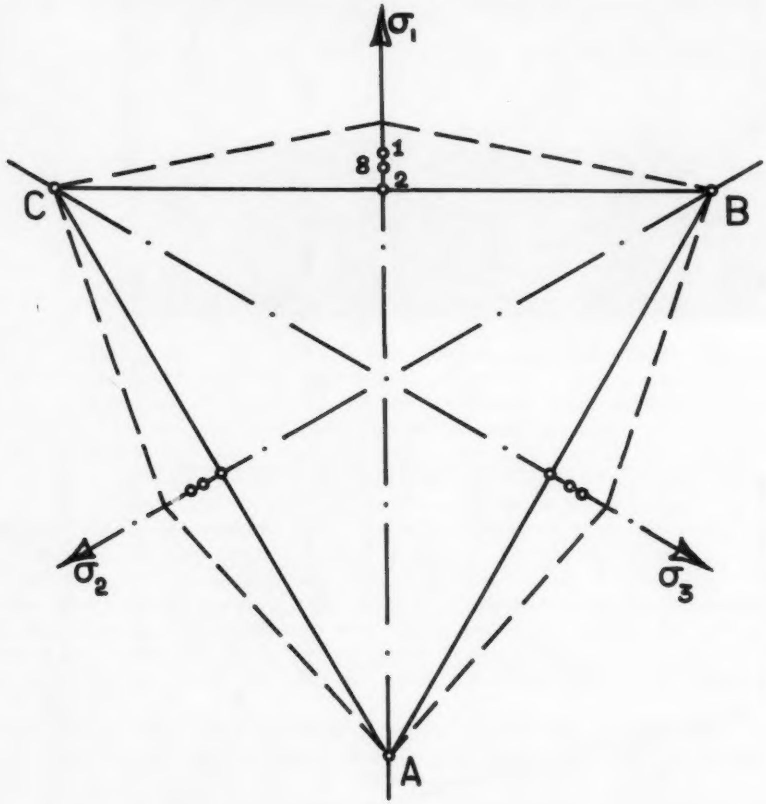

FIG. 13.—TRIAXIAL TESTS ON CRUSHED QUARTZ SAND (LANTERN HILL). PROJECTION OF TEST RESULTS ONTO THE PLANE $\sigma_1 + \sigma_2 + \sigma_3 = -1$. THE COULOMB YIELD CRITERION IS SHOWN AS THE DOTTED LINE.

in the case of material with small cohesion the strength is very nearly proportional to the mean stress, it is possible to represent the data on a single plane in this space. Fig. 13 shows data from the present series of tests projected onto the plane

$$\sigma_1 + \sigma_2 + \sigma_3 = 1 \qquad (44)$$

For comparison, a hexagon representing the Coulomb yield criterion has been drawn through the compression test data. The extension test results lie well within this hexagon, so that in the light of this alternative interpretation the data appears to be inconsistent with a yield criterion of the generalized Coulomb type.

AN ALTERNATIVE YIELD CRITERION

The Coulomb concept of a soil as a material with internal friction is based on the observed linearity between the normal stress on a plane and the shear stress required to induce shear strain in that plane. The two empirical constants defining the linear relationship are called the cohesion and the angle of friction, by analogy with the case of a block sliding on a surface. This analogy cannot, of course, be pursued too far. For example, it is well known that the effective angle of friction is dependent on the shape of the soil particles and the manner of packing as well as on the angle of friction at the points of contact. There is also no evidence that the angle of friction is genuinely independent of the direct stress components parallel to the plane of slip. Furthermore, there seems to be no physical argument which places the Coulomb yield criterion in a preferred position in relation to other possible criteria for combined stress states. The justification for its adoption is rather that it is simple in concept and has been consistent with the available experimental evidence.

The experimental results presented herein indicate that estimates of yield stress under combined stress that are based on the Coulomb yield criterion may overestimate the actual yield strength appreciably. If this evidence has been interpreted correctly, we are led to conclude that use of the Coulomb yield criterion to extend triaxial test results to other stress states might under certain circumstances lead to an overestimate of the failure load of a soil mass. Is it possible to substitute a criterion that would enable conservative estimates of failure loads to be made on the basis of triaxial test results, irrespective of the actual stress distribution in the soil? If soil is accepted as an ideally plastic material, use may be made of the result that yield surfaces that lie inside the actual yield surface in principal stress space will meet this requirement (22). Since the form of the yield criterion is unknown this leads to a search for the smallest yield surface that can be drawn through the triaxial test data. If the concept of stability is retained—a necessity if the discussion of the problem is to be continued on the basis of plastic theory—the yield surface must be convex (see Introduction) and so it cannot lie inside a triangle drawn through the compression triaxial test data, Fig. 13. The figure with this triangle as cross section then represents a yield criterion, that meets the stated requirements.

The case of a material with both cohesion and internal friction will be taken when establishing formulae for the yield surface. The generators through points B and C, Fig. 13, have equations of the type of Eq. 3b. Side BC lies in a plane containing these two lines and its equation will be

$$\sigma_1 = \frac{\sigma_2 + \sigma_3}{2} (1 - \sin \phi) + c \cos \phi, \quad (\sigma_2, \sigma_3 \leqslant \sigma_1) \quad \ldots \quad (45a)$$

The other two planes forming the surface are

$$\sigma_2 = \frac{\sigma_3 + \sigma_1}{2} (1 - \sin \phi) + c \cos \phi, \quad (\sigma_3, \sigma_1 \leqslant \sigma_2) \quad \ldots \quad (45b)$$

and

$$\sigma_3 = \frac{\sigma_1 + \sigma_2}{2} (1 - \sin \phi) + c \cos \phi, \quad (\sigma_1, \sigma_2 \leqslant \sigma_3) \quad . . . \quad (45c)$$

Eqs. 45 define the yield surface in principal stress space in terms of the constants c and ϕ as determined from the compressive triaxial test.

The plane stress cross section is a triangle with apexes at the points B, D, and F in Fig. 4. It is evident that, in the case of the compression test, the velocity fields used to obtain complete solutions for the Coulomb criterion can also be used for the new criterion, with similar results regarding uniqueness of the surface tractions. For the extension test, the corresponding velocity fields cannot be used; however, it is a simple matter to establish that a velocity field corresponding to a homogeneous strain can be associated with the stress distribution and hence uniqueness of surface tractions is confirmed in this case also. Details will not be given herein.

Turning now to the test data shown in Fig. 10, a comparison of Eqs. 19 and 45c shows that the curve marked $\sigma_\theta = \sigma_z$ in Fig. 10 also represents the prediction of the new yield criterion. The test points lie below this curve, and hence outside the new yield surface. Thus the use of the new yield criterion will lead to conservative estimates of the carrying capacity for all the sands for which experimental data has been quoted.

CONCLUSION

The analysis of the previous section establishes that a wide range of ratios of axial to lateral pressure is possible in the triaxial extension test without it becoming necessary to infer that the stress state in the triaxial specimen has become non-homogeneous. To permit this interpretation, the usual generalization of the Coulomb yield criterion must be abandoned. An alternative yield condition can be constructed (Eqs. 45) corresponding to a surface in principal stress space lying entirely inside all the available data for sands. This particular surface also has the important property of lying inside all possible surfaces that can be drawn through the data for the standard triaxial compression test when the postulate of stability (and hence convexity of the yield surface) is retained. As a consequence of the latter property, use of the new criterion will result in lower bounds to the collapse load whatever the actual form of the yield surface in principal stress space. This means that it can be used with more confidence than the Coulomb criterion when the only experimental data available is that from the standard triaxial test.

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Since this work was completed, H. G. Hopkins has pointed out that the velocity fields (Eqs. 34 and 41) have appeared in a report of the British Armament Research and Development Establishment (23).

APPENDIX I

Uniqueness of applied surface forces when the yield surface is not strictly convex but contains flats.

Theorem.—If the prescribed surface velocities acting on a body composed of stable, ideally plastic material are uniform in magnitude and direction, then the component of the resultant surface force in the direction of motion of the surface is uniquely defined.

This theorem has been proved by Hill (11) for the more restrictive case of a strictly convex yield surface and by D. C. Drucker, H. J. Greenberg, E. H. Lee, and W. Prager (24) for a Prandtl-Reuss material in plane strain. The latter proof (which is based on the limit analysis theorems of Drucker, Prager and Greenberg (24)), can be extended to cover the case considered here.

Proof.—Consider a body for which the velocities u_i (i = 1, 2, 3) are specified over the region A_u and the tractions T_i (force per unit area) are specified over the region A_T of the surface. Suppose a solution with stresses σ_{ij} (i, j = 1, 2, 3), strain rates ϵ_{ij} and resultant surface force F_i on A_u, and a second solution with stresses $\sigma_{ij}{}^*$, etc. Suppose further that the two solutions possess velocity singularities only at the same points, so that the difference between the velocities always remains finite. Then by the principle of virtual work

$$\int (T_i - T_i{}^*)(u_i - u_i{}^*)\, dA = \int (\sigma_{ij} - \sigma_{ij}{}^*)(\epsilon_{ij} - \epsilon_{ij}{}^*)\, dV \quad \ldots . (46)$$

Now $u_i = u_i{}^*$ on A_u and $T_i = T_i{}^*$ on A_T, so the left-hand side of Eq. 46 vanishes. The flow rule for a stable material (10) requires that

$$(\sigma_{ij} - \sigma_{ij}{}^*)\, \epsilon_{ij} \geqslant 0 \quad \ldots\ldots\ldots (47a)$$

and also

$$(\sigma_{ij}{}^* - \sigma_{ij})\, \epsilon_{ij}{}^* \geqslant 0 \quad \ldots\ldots\ldots\ldots (47b)$$

so the constituent parts of the integrand on the right-hand side of Eq. 46 are non-negative; hence

$$\int (\sigma_{ij} - \sigma_{ij}{}^*)\, \epsilon_{ij}\, dV = 0 \quad \ldots\ldots\ldots\ldots\ldots (48)$$

Applying the principle of virtual work again,

$$\int (T_i - T_i{}^*) u_i\, dA = \int (\sigma_{ij} - \sigma_{ij}{}^*)\, \epsilon_{ij}\, dV \quad \ldots\ldots (49)$$

Comparing Eqs. 48 and 49 and recalling that T_i = Ti* on A_T,

$$\int_{A_u} (T_i - T_i{}^*)\, u_i\, dA = 0 \quad \ldots\ldots\ldots\ldots\ldots (50)$$

The velocities u_i are constant in magnitude and direction on A_u, so

$$F_i \; u_i = F_i{}^* u_i \quad \text{.................} \quad (51)$$

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APPENDIX III.—NOTATION

The following symbols, adopted for use in the paper and for the guidance of discussers, conform essentially with "Glossary of Terms and Definitions in Soil Mechanics," prepared by the Committee on Glossary of Terms and Definitions in Soil Mechanics of the Soil Mechanics and Foundations Division, Proceedings Paper 1826, October, 1958:

A = area;

a = radius of triaxial test specimen;

c = cohesion per unit area;

D = rate of energy dissipation;

F = resultant surface force;

k = $(\sigma_\theta - \sigma_z)/(\sigma_r - \sigma_z)$;

N = $\tan(\pi/4 + \phi/2)$;

p = lateral pressure in the triaxial test;

Q	=	excess axial load in the triaxial test;
$\bar{q}$	=	mean axial pressure in the triaxial test;
r, θ, z	=	cylindrical co-ordinates;
T	=	surface traction per unit area;
u	=	radial velocity;
V	=	volume;
w	=	axial velocity;
γ_{xy}	=	shear strain rate;
ϵ_1 etc.	=	direct strain rates;
λ	=	positive constant;
σ	=	stress;
τ	=	shear stress;
ϕ	=	angle of friction;
Ψ	=	angle (Fig. 8); and
Δ	=	dilatation of a triaxial test specimen per unit axial compression.

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GENERALIZED SOLUTIONS FOR LATERALLY LOADED PILES

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SYNOPSIS

To reach rational solutions for problems of laterally loaded piles, the non-linear force-deformation characteristics of the soil must be considered. This may be done by repeated application of elastic theory. Soil modulus constants are adjusted for each successive trial until satisfactory compatibility is obtained in the structure-pile-soil system. The computations are facilitated by non-dimensional solutions.

Basic equations and methods of computation are given for both elastic-pile theory and rigid-pile theory. Several forms of soil modulus variation with depth are considered. Typical solutions are presented and recommendations given for their use in design problems.

INTRODUCTION

The problem of the laterally loaded pile is of particular interest in connection with drilling platforms and other offshore pile-supported industrial and defense installations. Lateral loads from wind and wave are frequently the most critical factor in the design of such structures. Solutions of the general problem also apply to a variety of cases onshore, including power poles, pile-supports for earthquake resistant structures, and pile-supported structures which may be subjected to lateral blast forces.

The problem of laterally loaded piles is closely related to the familiar problem of a beam on an elastic foundation; however, in one respect, it represents a more specialized case. All external forces and moments applied to

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the pile-soil system are introduced through boundary conditions existing at one point, the top of the pile, while loading may be applied at many points along a beam. On the other hand, rational solutions of pile-soil interaction problems require generalization of the beam-on-elastic-foundation theory to account for the non-linear characteristics of real soils.

To account for the non-linearity between pile deflection and soil resistance, the most convenient approach appears to be one in which repeated application of elastic theory is used. Soil resistance moduli are adjusted upon completion of each trial until satisfactory compatibility is obtained between the predicted behavior of the soil and the load-deflection relationships required by an elastic pile.

In the final trial in a series of iterative approximations the final variation in soil modulus may assume any form with respect to distance along the pile. If it is required that the soil modulus values be adjusted independently at each depth considered, then, in practice, a rigorous solution for any particular pile problem requires the use of a digital computer.

Fortunately, the final computed deflections, bending moments, and other quantities are not very sensitive to changes in soil modulus values. Satisfactory results may be obtained for most practical cases with simple forms of soil modulus variation with depth. Adjustments are limited to changes in the values of the coefficients of soil modulus variation.

In most cases, for both clay and sandy soils, the final soil modulus values tend to increase with depth. The principal reasons for this are that (1) soils frequently increase in strength characteristics with depth as the result of overburden pressures and of natural deposition and consolidation processes and (2) pile deflections decrease with depth for any given loading, and the corresponding equivalent elastic moduli of soil reaction tend to increase with decreasing deflection.

Non-dimensional solutions have been presented previously[3] in which the soil modulus, E_s, increases in simple proportion to depth x, or $E_s = k\,x$. While this simple form appears to be applicable to most laterally loaded pile problems, a few cases have been encountered where it would be helpful to use some other soil modulus function. For example, in the interpretation of the results of several series of extensive field tests with an instrumented pile[4,5] other forms have been found desirable. With design problems, however, the uncertainty inherent in estimating soil behavior characteristics from conventional soil tests is usually consistent with the small errors which may be introduced by the use of a simple form of soil modulus-depth function, such as $E_s = k\,x$.

The purpose of the present paper is to consider general solutions for laterally loaded piles which are supported by an elastic medium. Derivations

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[3] "Non-Dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth," by Lymon C. Reese, Proceedings, Eighth Texas Conf. on Soil Mechanics and Foundation Engrg., Special Publ. No. 29, Bur. of Engrg. Res., The Univ. of Texas, Austin, Tex., September, 1956.

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and methods are given by which non-dimensional solutions may be computed for any desired form of variation of soil modulus with respect to depth.

Notation.—The letter symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically, for convenience of reference, in Appendix II.

STATEMENT OF PROBLEM

The soil modulus is defined as

$$E_s = \frac{-p}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which y is the lateral deflection of the pile and p is the soil resistance expressed as force per unit length of pile. The negative sign indicates that the direction of the soil reaction is always opposite to the direction of the pile deflection. The soil reaction need not be a linear function of pile deflection and in general is not. A typical relation between p and y is shown in Fig. 1. The soil modulus, E_s, is the slope of a secant drawn from the origin to any point along the p - y curve. Its units are force $\times$ length^{-2}.

Soil reaction-pile deflection relations have been the subject of field and analytical research studies;[6,7,8,9] recommendations have been given for predicting p - y relations from the results of soil investigations,[6,10] and other research is in progress. In time, most of this information probably will be available to the engineering profession.

A typical foundation pile of length, L, and flexural stiffness, E I, is shown in Fig. 2(a). The depth, x, is measured downward from the ground line. In this example, the boundary condition at the top consists of an imposed moment, M_t, and a shear, P_t, and each is shown acting in a positive sense. Other combinations of boundary values would be applicable in variations of this problem.

The variation in soil modulus with depth, corresponding to specific values of the loading P_t and M_t, is indicated in Fig. 2(b). If the loads change in value, a different deflection pattern will be taken by the pile and different values of the soil modulus will result. Considering the non-linearity of p - y relations at various depths, E_s is a function of both x and y. Therefore, the form of the E_s versus depth relationship also will change if the loading is changed. However, it may be assumed temporarily (subject to adjustment of E_s values by trial and error) that the soil modulus is some function of x only, or that

$$E_s = E_s(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For solution of the problem the elastic curve y (x) of the pile, shown in Fig. 2(c), must be determined, together with various derivatives which are of interest. These derivatives yield values of slope, moment, shear, and soil re-

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6 "Soil Modulus for Laterally Loaded Piles," by Bramlette McClelland and John A. Focht, Jr., Transactions, ASCE, Vol. 123, 1958, p. 104.

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action as functions of depth. The successive curves are shown in Fig. 3 for a typical pile problem. The sign conventions are shown in Fig. 4.

DIMENSIONAL ANALYSIS FOR ELASTIC-PILE THEORY

The principles of dimensional analysis may be used to establish the form of non-dimensional relations for the laterally loaded pile. With the use of model

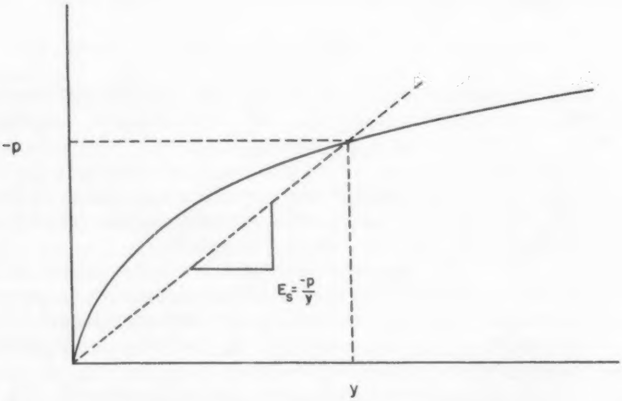

FIG. 1.—TYPICAL RELATION BETWEEN SOIL REACTION p AND PILE DEFLECTION y

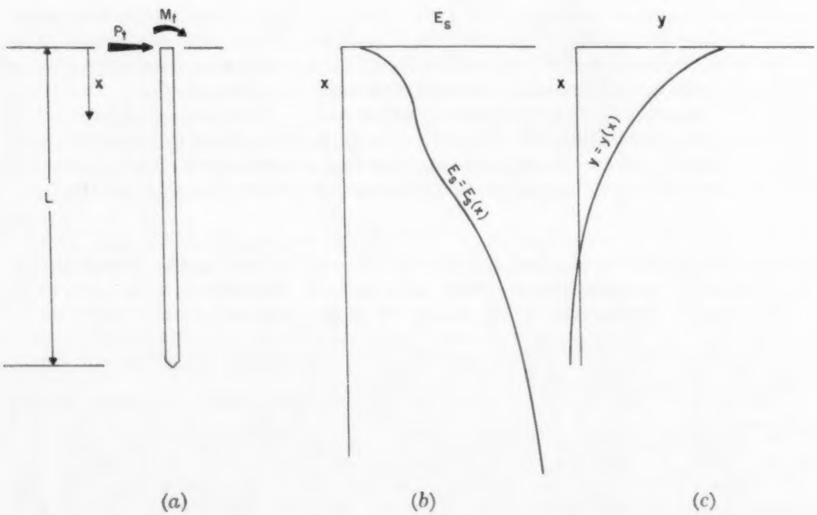

FIG. 2.—A LATERALLY LOADED PILE PROBLEM AND ITS SOLUTION

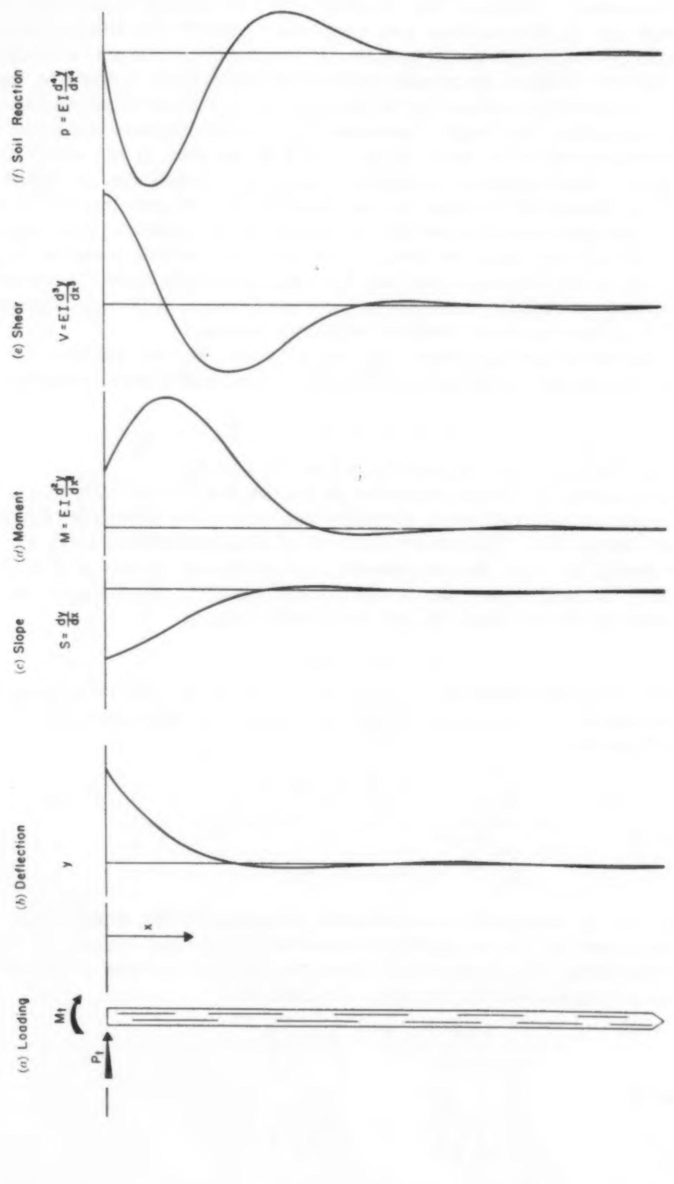

FIG. 3.—COMPLETE RESULTS FROM A PILE SOLUTION

theory the necessary relations will be determined between a "prototype" having any given set of dimensions, and a similar "model" for which a solution may be available. Although the principles of dimensional analysis are usually applied to results obtained from tests on physical models, the method is equally applicable to results obtained by the solution of a mathematical model.

For very long piles, the length, L, loses significance because the deflection may be very nearly zero for much of the length of the pile. It is convenient to introduce some characteristic length dimension as a substitute. A linear dimension, T, is therefore included in the quantities to be considered. T may be defined in any convenient way which will simplify the notation. The specific definition of T will vary with the form of the function for soil modulus versus depth. However, it will be seen later that, for each definition used, T expresses a relation between the stiffness of the soil and the flexural stiffness of the pile. It is therefore given the term "relative stiffness factor."

For the case of an applied shear, P_t, and moment, M_t, the solution for deflections of the elastic curve may include the relative stiffness factor and be expressed as

$$y = y\left(x, T, L, E_s, E\,I, P_t, M_t\right) \ldots\ldots\ldots\ldots (3)$$

Other boundary values could be substituted for P_t and M_t.

If the assumption of elastic behavior is introduced for the pile, and if deflections remain small, relative to the pile dimensions, the principle of superposition may be applied. Thus the effects of an imposed lateral load, P_t, and imposed moment, M_t, may be considered separately, as shown in Fig. 5. If y_A represents the deflection due to the lateral load, P_t, and if y_B is the deflection caused by the moment, M_t, the total deflection is

$$y = y_A + y_B \qquad (4)$$

Furthermore, it is the ratios of y_A to P_t and of y_B to M_t which are sought in reaching generalized elastic-case solutions. Thus, the solutions may be expressed for Case A as

$$\frac{y_A}{P_t} = f_A\left(x, T, L, E_s, E\,I\right) \ldots\ldots\ldots\ldots (5)$$

and for Case B

$$\frac{y_B}{M_t} = f_B\left(x, T, L, E_s, E\,I\right) \ldots\ldots\ldots\ldots (6)$$

in which f_A and f_B represent two different functions of the same terms. In each case there are six terms and two dimensions (force and length) involved. There are, therefore, four independent, non-dimensional groups which can be formed. The arrangements chosen are, for Case A,

$$\frac{y_A\,E\,I}{P_t\,T^3}, \frac{x}{T}, \frac{L}{T}, \frac{E_s\,T^4}{E\,I} \ldots\ldots\ldots\ldots (7)$$

and for Case B,

$$\frac{y_B\,E\,I}{M_t\,T^2}, \frac{x}{T}, \frac{L}{T}, \frac{E_s\,T^4}{E\,I} \ldots\ldots\ldots\ldots (8)$$

To satisfy conditions of similarity, each of these groups must be equal for both model and prototype, or,

$$\frac{x_p}{T_p} = \frac{x_m}{T_m} \ldots\ldots\ldots\ldots (9)$$

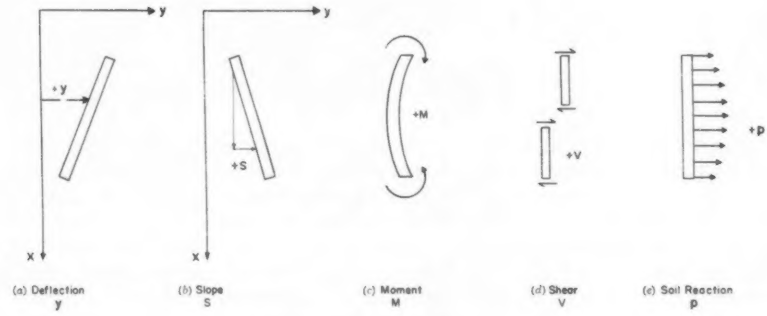

FIG. 4.—SIGN CONVENTIONS

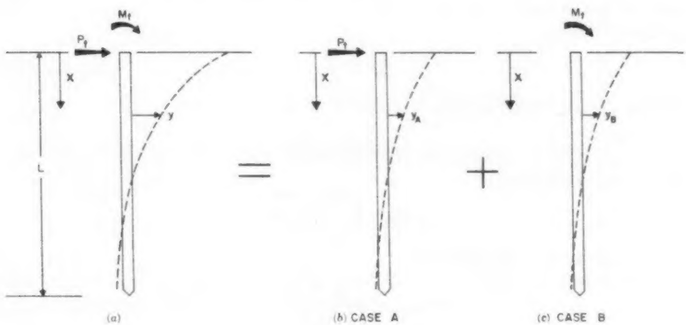

FIG. 5.—APPLICATION OF THE PRINCIPLE OF SUPERPOSITION TO THE LATERALLY LOADED PILE PROBLEM

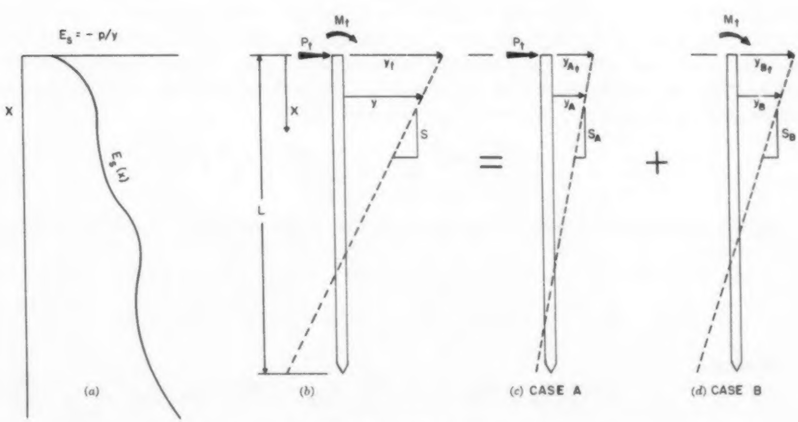

FIG. 6.—THE PROBLEM OF A SHORT, RIGID PILE

$$\frac{L_p}{T_p} = \frac{L_m}{T_m} \quad \text{.................... (10)}$$

$$\frac{E_{s_p}\, T_p^{4}}{E\, I_p} = \frac{E_{s_m}\, T^{4}}{E\, I_m} \quad \text{............... (11)}$$

$$\frac{y_{A_p}\, E\, I_p}{P_{t_p}\, T_p^{\,2}} = \frac{y_{a_m}\, E\, I_m}{P_{t_m}\, T_m^{\,3}} \quad \text{.............. (12)}$$

and

$$\frac{y_{B_p}\, E\, I_p}{M_{t_p}\, T_p^{2}} = \frac{y_{B_m}\, E\, I_m}{M_{t_m}\, T_m^{\,2}} \quad \text{.............. (13)}$$

A group of non-dimensional parameters may be defined which will have the same numerical value for any pair of structurally similar cases, or for any model and its prototype. These are

Depth Coefficient,

$$Z = \frac{x}{T} \quad \text{.................... (14)}$$

Maximum Depth Coefficient,

$$Z_{max} = \frac{L}{T} \quad \text{................... (15)}$$

Soil Modulus Function,

$$\phi\,(Z) = \frac{E_s\, T^4}{E\, I} \quad \text{................. (16)}$$

Case A Deflection Coefficient,

$$A_y = \frac{y_A\, E\, I}{P_t\, T^3} \quad \text{.................. (17)}$$

Case B Deflection Coefficient,

$$B_y = \frac{y_B\, E\, I}{M_t\, T^2} \quad \text{.................. (18)}$$

Thus, from Eqs. 14 through 18, for (1) similar soil-pile stiffness systems, (2) similar positions along the piles, and (3) similar pile lengths (unless lengths are very great and need not be considered), the solution of the problem can be expressed from Eq. 4, and from Eqs. 17 and 18, as

$$y = \left[\frac{P_t\, T^3}{E\, I}\right] A_y + \left[\frac{M_t\, T^2}{E\, I}\right] B_y \quad \text{.......... (19)}$$

By the same type of reasoning other forms of the solution can be expressed as

Slope,

$$S = S_A + S_B = \left[\frac{P_t\, T^2}{E\, I}\right] A_s + \left[\frac{M_t\, T}{E\, I}\right] B_s \quad \text{........ (20)}$$

Moment,

$$M = M_A + M_B = \left[P_t\ T\right] A_m + \left[M_t\right] B_m \quad \text{........ (21)}$$

Shear,

$$V = V_A + V_B = \left[P_T\right] A_v + \left[\frac{M_t}{T}\right] B_v \qquad (22)$$

Soil Reaction,

$$p = p_A + p_B = \left[\frac{P_t}{T}\right] A_p + \left[\frac{M_t}{T^2}\right] B_p \qquad (23)$$

It is still necessary to obtain a particular set of A and B coefficients (as functions of the depth parameter Z) by a solution of a particular model. However, Eqs. 20 through 23 are independent of the characteristics of the model except that elastic behavior and small deflections are assumed. Note that T is still an undefined characteristic length dimension and that the variation of E_s with depth, or the corresponding form of ϕ (Z), has not been specified.

The relations previously derived are applicable to step-tapered piles which are conventionally used in offshore construction. However, it is necessary that structural similarity be maintained between the mathematical model and the prototype. This means that the changes in E I must be proportionate and must occur at corresponding values of Z along the lengths of both model and prototype.

THE DIFFERENTIAL EQUATION

From beam theory, the basic equation for an elastic beam is

$$E\,I \frac{d^4y}{dx^4} = p \qquad (24)$$

Introducing the definition of $p = -E_s\,y$ of Eq. 1, the basic equation for a beam on an elastic foundation, or for a laterally loaded pile, is

$$\frac{d^4y}{dx^4} + \frac{E_s}{E\,I}\,y = 0 \qquad (25)$$

Where an applied lateral load, P_t, and an applied moment, M_t, are considered separately according to the principal of superposition, Eq. 25 becomes, for Case A,

$$\frac{d^4y_A}{dx^4} + \frac{E_s}{E\,I}\,y_A = 0 \qquad (26)$$

and for Case B,

$$\frac{d^4y_B}{dx^4} + \frac{E_s}{E\,I}\,y_B = 0 \qquad (27)$$

Substituting the definitions of non-dimensional parameters contained in Eqs. 14 through 18, a non-dimensional differential equation can be written for Case A as

$$\frac{d^4A_y}{dZ^4} + \phi\,(Z)\,A_y = 0 \qquad (28)$$

and for Case B,

$$\frac{d^4 B_y}{dZ^4} + \phi\,(Z)\, B_y = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (29)$$

SOLUTION OF THE DIFFERENTIAL EQUATIONS

To produce a particular set of non-dimensional A and B coefficients, it is necessary (1) to specify ϕ (Z), including a convenient definition of the relative stiffness factor, T, and (2) to solve the differential equations (Eqs. 28 and 29). The resulting A and B coefficients may then be used, with Eqs. 19 through 23, to compute deflections, slopes, moments, shears, and soil reactions for any pile problem which is similar to the case for which non-dimensional solutions have been obtained.

Based on the boundary conditions, P_t and M_t, and the resulting A and B coefficients, relations may be derived so that problems may be solved for cases in which other boundary conditions are known. These may include a specified deflection, y_t, at the top of the pile or a specified value of slope, S_t. Where structural restraints are involved at the pile-to-structure connection, the boundary conditions may consist of a lateral shear, P_t, and rotational restraint stiffness, M_t/S_t. It is conceivable that still other combinations and ratios may have application in variations of the problem.

Further non-dimensional coefficients could be obtained for these other boundary conditions. However, it appears that any conceivable structure-pile-soil system can be solved by use of the A and B coefficients. Some of these solutions involve a trial and error process in satisfying the boundary conditions.

The only case for which a closed analytical solution is possible is that in which the soil modulus is constant with depth.[11] Series-type solutions have also been developed for the case in which the soil modulus has a linear variation with depth.[12]

Since the available special-case solutions impose limitations on the nature of the soil modulus variation, a more general approach is desirable. The difference-equation method appears to offer the most practical approach to generalized solutions.[13] Procedures have been developed for a once-through computation with successive elimination of unknowns.[14,15] A further extension has been made[3] to enable the introduction of moment and shear as the boundary conditions, and to produce a set of non-dimensional solutions for $E_s = k\,x$.

A computer solution has been developed in which the flexural stiffness E I of the pile may be changed abruptly at points along the length of the pile.[16]

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15 "A Numerical Method for Predicting the Behavior of Laterally Loaded Piling," by R. J. Howe, TS Memorandum 9, Shell Oil Co., Houston, Tex., 1953.

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The changes in pile cross section ordinarily encountered in offshore construction produce relatively moderate effects in computed moments. However, the development of a limited number of non-dimensional solutions which include step-changes in E I, may be warranted for a few typical cases.

The difference-equation method is summarized in Appendix I, as it is applied to the case in which the pile stiffness, E I, is constant. This summary includes the equations which are needed to introduce the applied shear, P_t, and moment, M_t, as the boundary conditions.

The summary in Appendix I is given in terms of ordinary parameters having physical dimensions. To produce a set of non-dimensional solutions for any desired soil modulus function, it is possible to use input values from a dimensioned "model" and then to determine the corresponding non-dimensional coefficients by Eqs. 14 through 23. However, values may be computed directly by (1) substituting unit values for P_t and M_t, (2) setting the k coefficients in the soil modulus function $E_s(x)$ to correspond to constants in ϕ (Z) (thus the relative stiffness factor, T, is made equal to unity), and (3) letting the length, L, of the pile be numerically equal to the desired value of Z_{max}. The various results will then be numerically equal to the corresponding non-dimensional coefficients.

LIMITING CASE FOR SHORT RIGID PILES

Piles or posts having relatively shallow imbedment are frequently encountered in practice. Such piles tend to behave as rigid members, and the difference-equation method used in the elastic-theory solutions tends to become inaccurate because of the very small successive differences which are involved. For such cases, a simpler theory is applicable, in which the pile is considered to be a rigid member.

Although computations are much more simple for the rigid-pile case than for the elastic-pile theory, it is still convenient to use generalized solutions and to consider separately the effects of applied lateral load and applied moment. This is particularly true where repeated solutions must be made to obtain compatibility between the behavior of the pile and the predicted characteristics of the soil.

The statement of the rigid-pile problem is illustrated in Fig. 6. For convenience, the notation and format of the development are made to correspond as closely as possible to those of the elastic-pile case.

DIMENSIONAL ANALYSIS FOR RIGID PILES

The pertinent factors in the rigid-pile problem and the principle of superposition are shown in Fig. 6. It is convenient to include an additional term, J, which is later given particular definitions which depend on the form of the soil modulus variation with depth. For the present, J is simply a constant having the same dimensions $\left(\text{force} \times \text{length}^{-2}\right)$ as the soil modulus, E_s.

For either Case A $\left(M_t = 0\right)$ or Case B $\left(P_t = 0\right)$ there are a total of six factors to be considered. For Case A,

$$y_A = y_A \left(x, L, E_s, J, P_t\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (30)$$

and for Case B,

$$y_B = y_B\left(x,\ L,\ E_s,\ J,\ M_t\right) \ldots\ldots\ldots\ldots \quad (31)$$

In each trial computation in an actual design problem, the soil is considered to be elastic. Thus, for either Case A or Case B, it is the ratio of deflection to loading which is sought in reaching generalized solutions. This reduces the number of non-dimensional groups to three. For Case A these are

$$\frac{y_A\ J\ L}{P_t},\ \frac{x}{L},\ \frac{E_s}{J} \ldots\ldots\ldots\ldots \quad (32)$$

and for Case B,

$$\frac{y_B\ J\ L^2}{M_t},\ \frac{x}{L},\ \frac{E_s}{J} \ldots\ldots\ldots\ldots \quad (33)$$

For similarity between a prototype and a computed model, each non-dimensional group may be defined as a dimensionless parameter. These are as follows:

Depth Coefficient,

$$h = \frac{x}{L} \ldots\ldots\ldots\ldots \quad (34)$$

Soil Modulus Function,

$$\phi\ (h) = \frac{E_s}{J} \ldots\ldots\ldots\ldots \quad (35)$$

Case A Deflection Coefficient,

$$a_y = \frac{y_A\ J\ L}{P_t} \ldots\ldots\ldots\ldots \quad (36)$$

Case B Deflection Coefficient,

$$b_y = \frac{y_B\ J\ L^2}{M_t} \ldots\ldots\ldots\ldots \quad (37)$$

By superposition, the total deflection $y = y_A + y_B$, is

$$y = \left[\frac{P_t}{J\ L}\right] a_y + \left[\frac{M_t}{J\ L^2}\right] b_y \ldots\ldots\ldots\ldots \quad (38)$$

From reasoning similar to the preceding, other forms of the solution can be expressed as:

Slope,

$$S = S_A + S_B = \left[\frac{P_t}{J\ L^2}\right] a_s + \left[\frac{M_t}{J\ L^3}\right] b_s \ldots\ldots \quad (39)$$

Moment,

$$M = M_A + M_B = \left[P_t\ L\right] a_m + \left[M_t\right] b_m \ldots\ldots \quad (40)$$

Shear,

$$V = V_A + V_B = \left[P_t\right] a_v + \left[\frac{M_t}{L}\right] b_v \ldots\ldots\ldots \quad (41)$$

Soil reaction,

$$p = p_A + p_B = \left[\frac{P_t}{L}\right] a_p + \left[\frac{M_t}{L^2}\right] b_p \ldots\ldots\ldots \quad (42)$$

For any given problem the slope $S = dy/dx$ is a constant and all higher derivatives of y are zero. The last three expressions are related to the first two through the relation between soil reaction and pile deflection, $E_s = -p/y$ or, in terms of the non-dimensional coefficients,

$$\phi\ (h) = \frac{-a_p}{a_y} = \frac{-b_p}{b_y} \ldots\ldots\ldots\ldots \quad (43)$$

The preceding dimensional analysis will apply to any form of the soil modulus functions E_s or ϕ (h). The soil modulus constant, J, is to be defined subsequently.

The non-dimensional soil modulus function, ϕ (h), is equivalent to the corresponding function, ϕ (Z), used with the elastic-pile theory except that ϕ (h) is related to the length of the pile rather than to a relative stiffness between the pile and the soil.

For any given ϕ (h), there exists a single set of non-dimensional coefficient curves (for deflection, slope, moment, shear, and soil reaction). Design problems may be solved by essentially the same procedures as for the elastic-pile case. The choice of which theory to use is aided by comparing the results of non-dimensional solutions obtained by the two methods.

EQUATIONS FOR RIGID-PILE SOLUTIONS

The equation for deflection y of a rigid pile is

$$y = y_t + S\,x \qquad (44)$$

in which y_t is the deflection at x = 0 and S is the constant slope of the pile. The soil reaction $p = - E_s\, y$ is

$$p = - E_s\, y_t - E_s\, S\, x \qquad (45)$$

A free body of the upper portion of a rigid pile is shown in Fig. 7 with all quantities shown acting in positive directions. By statics, the equation for shear is

$$V = P_t + \int_0^x p\, dx \qquad (46)$$

Substituting Eq. 45 into Eq. 46,

$$V = P_t - y_t \int_0^x E_s\, dx - S \int_0^x x\, E_s\, dx \qquad (47)$$

The equation for moment is

$$M = M_t + V\,x - \int_0^x x\, p\, dx \qquad (48)$$

or,

$$M = M_t + V\,x + y_t \int_0^x x\, E_s\, dx + S \int_0^x x^2\, E_s\, dx \qquad (49)$$

The shear and moment are zero at the bottom of the pile: at x = L, V = 0, and M = 0. Thus, Eqs. 50 and 51 may be written from Eqs. 47 and 49 so that y_t and S may be evaluated by simultaneous solution.

$$P_t = y_t \int_0^L E_s\, dx + S \int_0^L x\, E_s\, dx \qquad (50)$$

and

$$M_t = - y_t \int_0^L x\, E_s\, dx - S \int_0^L x^2\, E_s\, dx \qquad (51)$$

The values obtained for y_t and S are then substituted into Eqs. 47 and 49 to complete the solution.

As in the procedure used in the elastic-pile theory, unit values may be introduced into the solution to obtain numerically correct values of the non-dimensional coefficients defined in Eqs. 34 through 37. This amounts to de-

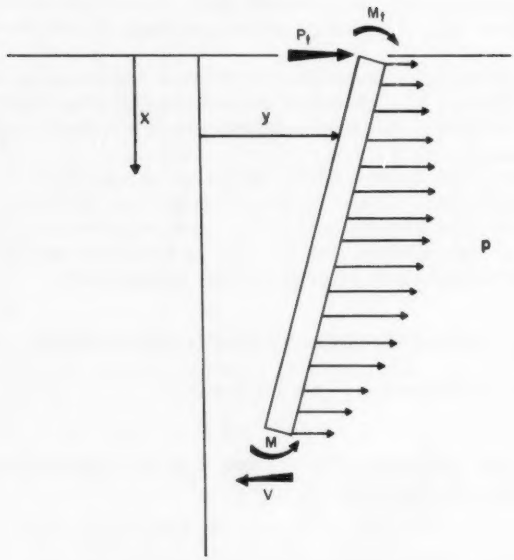

FIG. 7.—FREE BODY DIAGRAM FOR RIGID-PILE THEORY

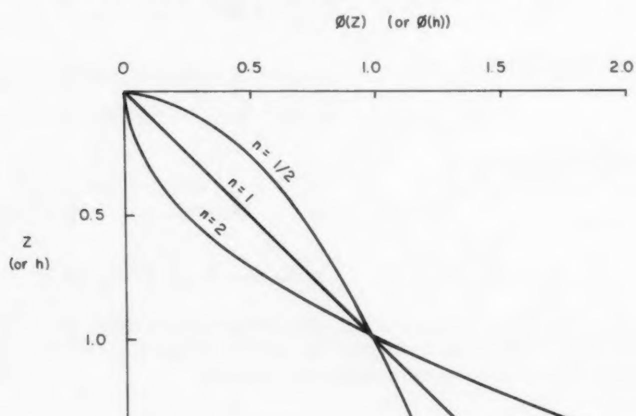

FIG. 8.—TYPICAL POWER FUNCTIONS FOR SOIL MODULUS

termining the non-dimensional coefficients from the results of a numerically convenient model having unit values of L, P_t, and M_t. Coefficients in E_s (x) are chosen to agree with those in the soil modulus function ϕ (h), and J is thus made equal to unity.

FORMS OF SOIL MODULUS VARIATION WITH DEPTH

In solving actual design problems of laterally loaded piles by use of previously computed non-dimensional solutions, the constants in the expressions describing the variation of soil modulus, E_s, with depth, x, are adjusted by trial until reasonable compatibility is obtained. The selected form of the soil modulus relation should be kept as simple as possible so that a minimum number of constants need be adjusted.

Two general forms which are adequate to express any continuous variation with depth are a power form,

$$E_s = k\,x^n \quad (52)$$

and a polynomial form,

$$E_s = k_o + k_1\,x + k_2\,x^2 \quad (53)$$

The form $E_s = k\,x$ is seen to be a special case of either of these. A form similar to Eq. 52 has been suggested previously.[17]

The relative stiffness factor, T, of the elastic-pile theory and the soil modulus constant, J, of the rigid-pile theory must be defined for each form of the soil modulus-depth relation. While they may be defined in any way, it is convenient to select definitions that will simplify the corresponding non-dimensional soil modulus functions.

POWER FUNCTIONS; $E_s = k\,x^n$

From the elastic-pile theory, Eq. 16 defining the non-dimensional soil modulus function is $\phi\,(Z) = \dfrac{E_s\,T^4}{E\,I}$.

If the form $E_s = k\,x^n$ is substituted in Eq. 16, the result is

$$\phi\,(Z) = \frac{k}{E\,I}\,x^n\,T^4 \quad (54)$$

For the elastic-pile case, it is convenient to define the relative stiffness factor, T, by the following expression.

$$T^{n+4} = \frac{E\,I}{k} \quad (55)$$

Substituting Eq. 55 into Eq. 54 gives

$$\phi\,(Z) = \frac{x^n\,T^4}{T^{n+4}} = \left[\frac{x}{T}\right]^n \quad (56)$$

---

17 "Piles Subjected to Lateral Thrust Part II - Analysis of Pressure, Deflection, Moment, and Shear by the Method of Difference Equations," by L. A. Palmer and P. P. Brown, Supplement of Symposium on Lateral Load Tests on Piles, ASTM Special Tech. Publication, No. 154-A, 1954, pp. 22-44.

Since $x/T = Z$, the general non-dimensional soil modulus function is

$$\phi(Z) = Z^n \quad (57)$$

Eq. 57 contains only one arbitrary constant, the power n. Therefore, for each value of n which may be selected, one complete set of independent non-dimensional solutions may be obtained from solution of Eqs. 28 and 29. For relatively short elastic piles, separate computations must be made for each Z_{max} considered.

From the rigid-pile theory the soil modulus function has been defined by Eq. 35 as $\phi(h) = E_s/J$. If the soil modulus constant, J, is now defined as

$$J = k L^n \quad (58)$$

the corresponding general non-dimensional soil modulus function is

$$\phi(h) = \frac{k x^n}{k L^n} \quad (59)$$

or, since $h = x/L$,

$$\phi(h) = h^n \quad (60)$$

Only one set of non-dimensional curves will be needed for each selected value of n, regardless of the length L.

In Fig. 8 several typical power functions are shown ($\phi = Z^n$ or $\phi = h^n$). The coordinates are used interchangeably for either the elastic-pile theory or the rigid-pile theory, although values of h greater than unity do not apply in the rigid-pile case.

POLYNOMIAL FUNCTION; $E_s = k_o + k_1 x + k_2 x^2$

When a polynomial is used to express the form of the soil modulus variation with depth, the relative stiffness factor, T, or the soil modulus constant, J, may be defined to simplify only one of the terms in the polynomial.

For the elastic-pile case, introducing the polynomial form into Eq. 16 gives

$$\phi(Z) = \frac{k_o T^4}{E I} + \frac{k_1 T^5}{E I}\left[\frac{x}{T}\right] + \frac{k_2 T^6}{E I}\left[\frac{x}{T}\right]^2 \quad (61)$$

To simplify the second term, as an example, T may be defined by the following expression:

$$T^5 = \frac{E I}{k_1} \quad (62)$$

The resulting soil modulus function is

$$\phi(Z) = r_o + Z + r_2 Z^2 \quad (63)$$

in which

$$r_o = \frac{k_o}{k_1}\left[\frac{1}{T}\right] \quad (64)$$

and

$$r_2 = \frac{k_2}{k_1}\left[T\right] \quad (65)$$

For the rigid pile theory, from Eq. 35,

$$\phi(h) = \frac{k_o}{J} + \frac{k_1 x}{J} + \frac{k_2 x^2}{J} \quad (66)$$

To again simplify the second term, J is defined by

$$J = k_1 L \qquad (67)$$

and

$$\phi(h) = \frac{k_o}{k_1 L} + \frac{k_1 x}{k_1 L} + \frac{k_2 x^2}{k_1 L} \qquad (68)$$

or

$$\phi(h) = r_o + h + r_2 h^2 \qquad (69)$$

in which $h = x/L$ and

$$r_o = \frac{k_o}{k_1 L} \qquad (70)$$

and

$$r_2 = \frac{k_2 L}{k_1} \qquad (71)$$

A separate set of non-dimensional curves would be needed for each desired combination of r-constants. Because of the complexity which otherwise would

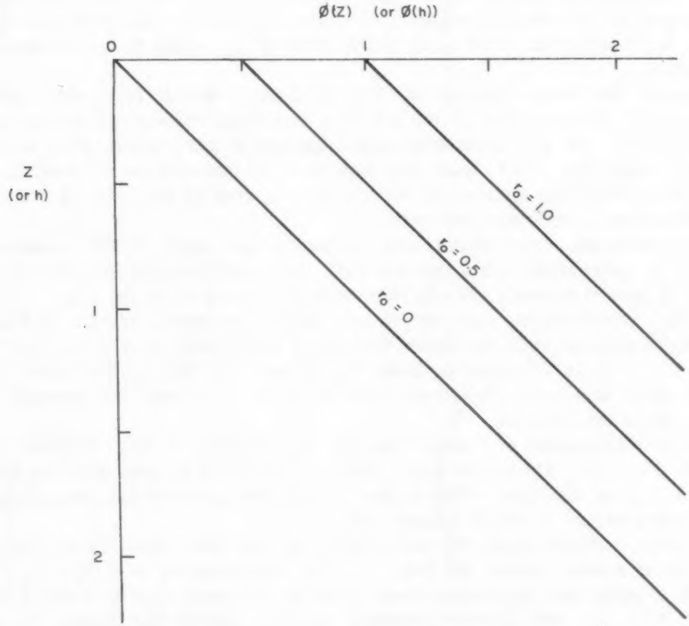

FIG. 9.—EXAMPLES OF SIMPLE POLYNOMIAL SOIL MODULUS FUNCTIONS

result, it does not appear reasonable to vary more than one constant and such forms as those following appear to be about as complicated as should be considered.

$$\phi(Z) = r_o + Z \qquad (72)$$

and

$$\phi(Z) = r_o + Z^2 \qquad (73)$$

Curves for the first form (Eq. 72) are shown in Fig. 9 $\left(\phi = r_o + Z \text{ or } \phi = r_o + h\right)$ for three values of r_o. From problems encountered in actual practice, the second form (Eq. 73) appears to offer interesting possibilities, but no solutions are included for this form.

While it would be permissible for some of the r-constants to have negative values, care must be taken that ϕ does not become negative. If this were to occur, some very peculiar results would be possible in the computed solutions.

TYPICAL SOLUTIONS FROM ELASTIC-PILE THEORY

The deflection and moment curves usually are the results of primary interest. These two quantities are used as a basis of comparison of various soil modulus functions in the results presented in the figures which follow. In most of these, results are shown separately for Case A $\left(M_t = 0\right)$ and Case B $\left(P_t = 0\right)$.

Fig. 10 shows the results of elastic-pile theory when various values of the exponent n are used in the form $E_s = k\ x^n$. The three values used correspond to the three curves of ϕ shown in Fig. 8. The solutions in Fig. 10 are applicable to very long piles with Z_{max} equal to approximately 5 or greater.

Although the three curves of Fig. 8 depart widely from each other at values of Z greater than about 1.0, the resulting deflections and moments shown in Fig. 10 are relatively much closer in agreement. This is in accordance with the fact that the behavior of the pile is related, through the relative stiffness factor, T, to the (n + 4) root of the ratio of pile stiffness, E I, to soil modulus constants.

The maximum deflections and moments are seen to increase as the value of n increases. This shows that the soil modulus values in the vicinity of Z less than unity clearly dominate the behavior of the pile.

Similar conclusions can be drawn from solutions shown in Fig. 11. The corresponding soil modulus functions are given in Fig. 9. The function ϕ (Z) = Z is common to both Fig. 10 and Fig. 11 and the corresponding deflection and moment curves may be used as a basis for comparisons between these two figures.

Greater variations are seen among the curves of Fig. 11 than among those in Fig. 10. There is also greater variation in soil modulus values near the top of the pile. This is further confirmation of the importance of soil modulus values at small values of Z.

Additional indication of the importance of the zone near the top is given by the comparison shown in Fig. 12. The constants r_1 in ϕ (Z) = r_1 Z are adjusted to give the same maximum positive moment coefficients as those obtained with the non linear function, ϕ (Z) = Z^n. In this figure, the solid curves have been reproduced from Figs. 8 and 10. They show the forms of soil modulus functions ϕ (Z) = $Z^{1/2}$ and ϕ (Z) = Z^2, and the corresponding moment coefficient curves for a long pile loaded by a lateral force, P_t, but with moment, M_t, equal to zero. The dashed lines in Fig. 12(a) are for soil modulus functions of the form $E_s = k\ x$. The coefficients of Z were determined to give the straight-line variations required to produce the same maximum positive moments as the two non-linear soil modulus functions. The values of Z at the intersections of the curved and straight-line variations are only 0.33 and 0.15 for the two examples con-

sidered. The corresponding moment curve comparisons are given in Fig. 12(b).

The comparisons in Fig. 12 suggest that good moment-curve predictions may be made by using $E_s = k\,x$ even though real soil modulus variations

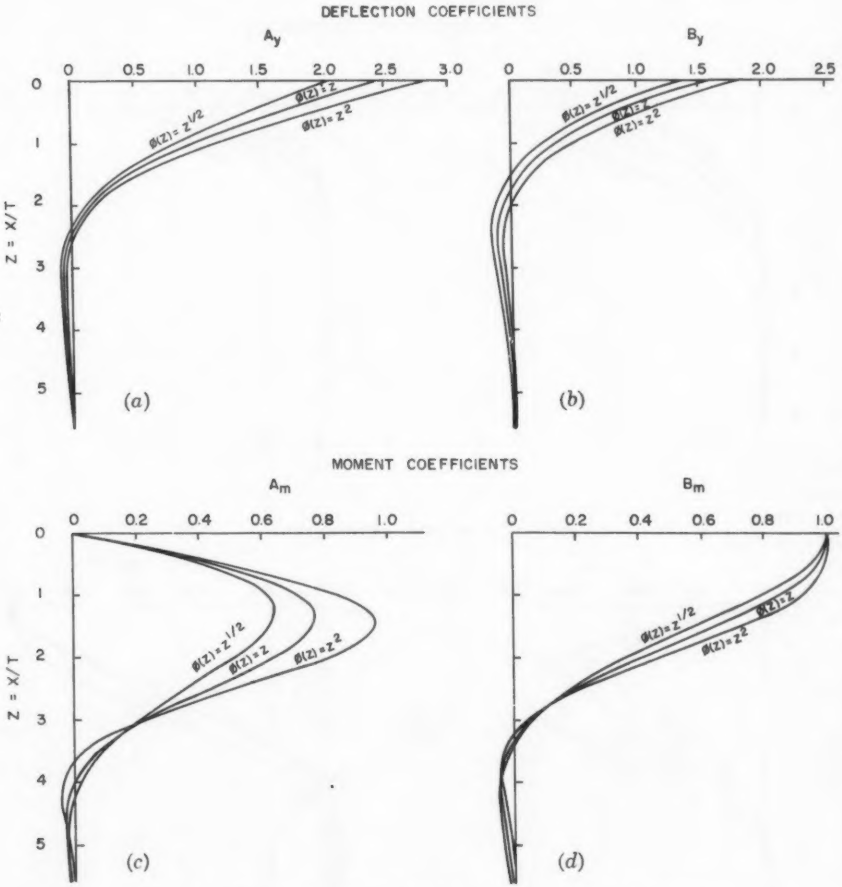

FIG. 10.—TYPICAL RESULTS OF ELASTIC-PILE THEORY WITH $E_s = k\,x^n$, or $\phi(Z) = Z^n$

may be quite non-linear with respect to depth. However, to make such satisfactory approximations, the designer must recognize the relatively great importance of close fitting very near the top of the pile.

TYPICAL SOLUTIONS FROM RIGID-PILE THEORY

The three soil modulus functions shown in Fig. 8 have been used to compute the behavior of a rigid pile also. The resulting deflection and moment curves for both Case A and Case B are shown in Fig. 13.

These results would be applicable to the case of a post or spud pile installed to relatively shallow depths. However, essentially the same trial and error design procedures may be followed as are used for reaching compatibility in the elastic-theory approach. As compared to the theory

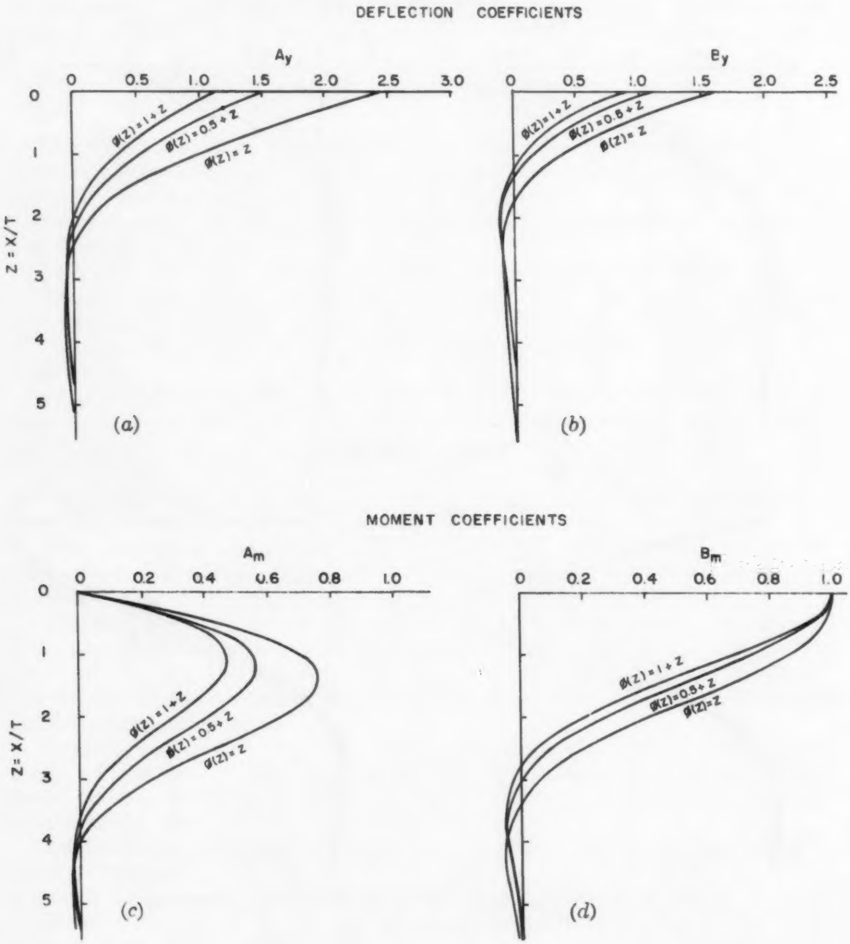

FIG. 11.—TYPICAL RESULTS OF ELASTIC-PILE THEORY WITH $E_s = k_o + k_1 x$, or, $\phi(Z) = r_o + Z$

for very long elastic piles, the major difference is that, for any given soil conditions, the final soil modulus function is related to the length of the pile.

The variations among the moment curves given in Fig. 13 are approximately consistent with those previously shown in Fig. 10 for the elastic-pile theory. However, these elastic-theory results were based on solutions for very long piles and direct comparison between these two figures has limited significance.

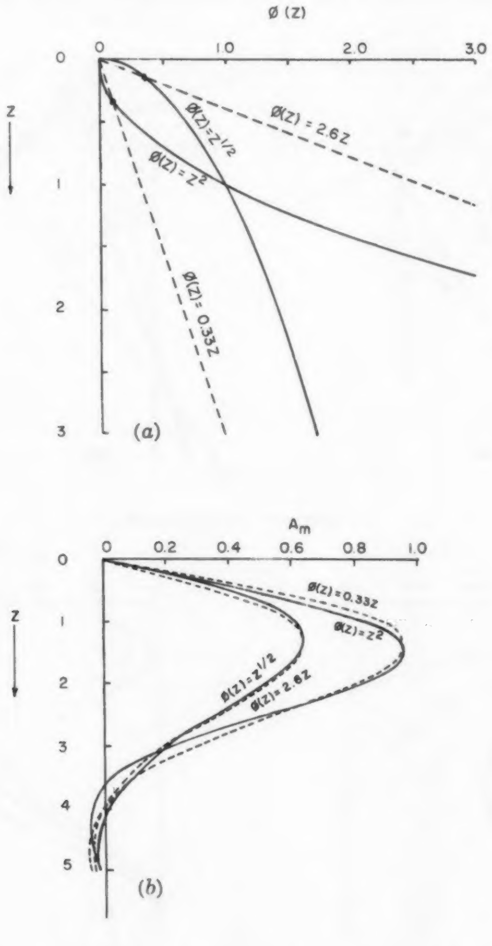

FIG. 12.—COMPARISON OF MOMENT VALUES OF $E_s = k\,x$ and $E_s = k\,x^n$

Considerable variation may be noted in Fig. 13 among the deflection curves for the rigid pile. Examination of the basic equations will show that, for any given length, L, the deflection and slope of the pile will vary accord-

ing to quantitative variations in Es; but that the moment, shear, and soil reaction depend only on the form of the soil modulus function.

EFFECT OF PILE LENGTH

Results obtained from elastic-pile theory for piles having Z_{max} values of 2, 3, 4, and 10 are shown in Fig. 14 by solid curves. These curves are based

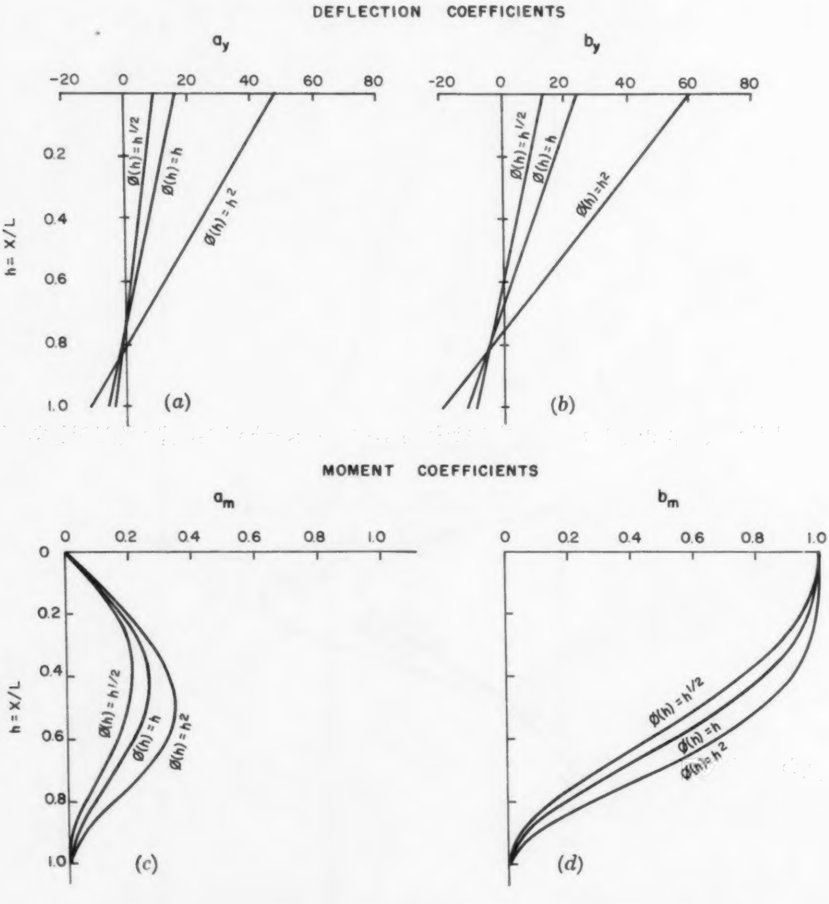

FIG. 13.—TYPICAL SOLUTIONS FROM RIGID-PILE THEORY WITH $E_s = k\,x^n\ \left(\phi(Z) = Z^n\right)$

on a soil modulus which increases in simple proportion to depth, or $E_s = k\,x$ ($\phi = Z$ or $\phi = h$). The corresponding non-dimensional function is $\phi\ (Z) = Z$. Results for the longest pile $\left(Z_{max} = 10\right)$ have been shown in previous figures.

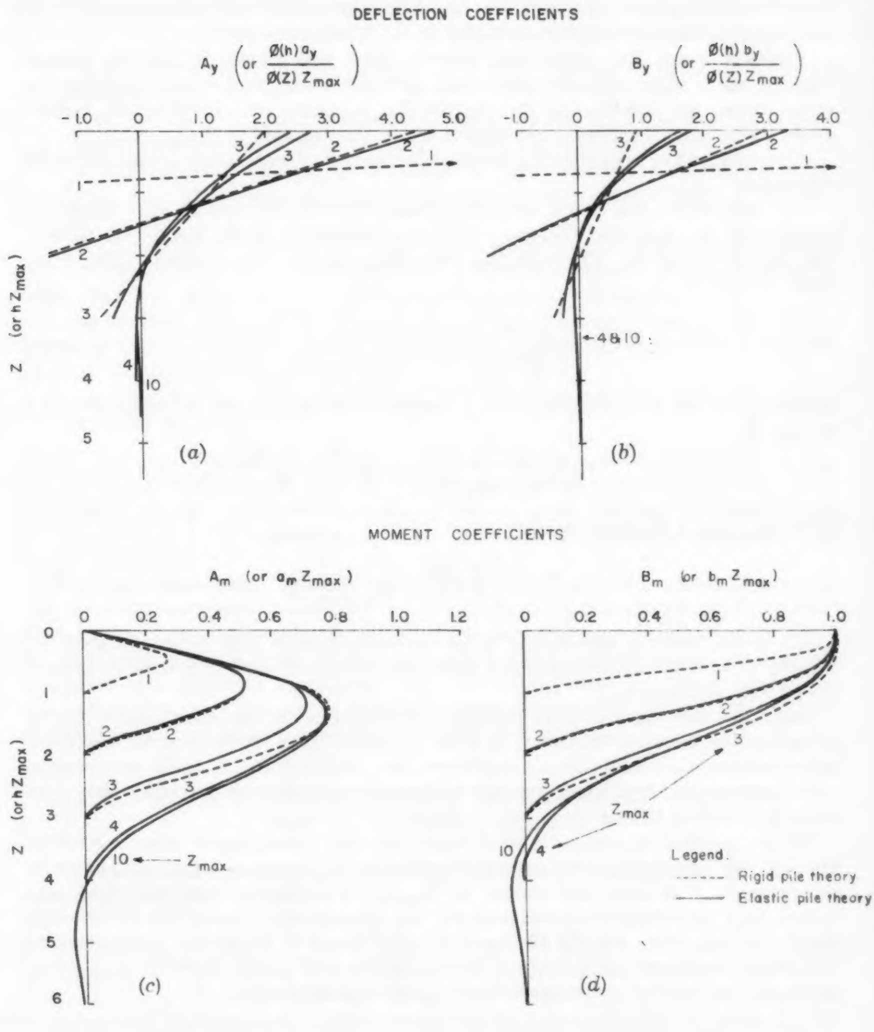

FIG. 14.—EFFECT OF PILE LENGTH AND COMPARISON OF RIGID-PILE THEORY WITH ELASTIC-PILE THEORY

The close agreement between the curves for $Z_{max} = 4$ and those for $Z_{max} = 10$ indicates that for all values of about 5 or greater, an elastic pile will act almost identically to one of infinite length.

The wide differences between the solid curves for $Z_{max} = 2$ and $Z_{max} = 3$ show that more solutions would be needed in this zone for actual design computations. Linear interpolation would be very inaccurate.

Results from the rigid-pile theory, also, are shown in Fig. 14. Dashed curves show rigid-pile deflection and moment coefficient values compared to those from the elastic-pile theory, for Z_{max} values of 2 and 3. Also, results from the rigid-pile theory are shown which correspond to $Z_{max} = 1$.

The method of adapting the rigid-pile results in explained by the following example.

To determine the basis of comparison between the elastic-pile deflection coefficient A_y and the corresponding coefficient a_y of the rigid-pile theory, the expressions for y_A from the two theories may be equated. Thus, from Eqs. 17 and 36,

$$\left[\frac{P_t\,T^3}{E\,I}\right] A_y = \left[\frac{P_t}{J\;L}\right] a_y \quad \text{. . . . . (74)}$$

Substituting the expression for E I contained in Eq. 16 and the definition of J of Eq. 35,

$$A_y\,P_t\,T^3\,\frac{\phi\,(Z)}{E_s\,T^4} = a_y\,\frac{P_t}{L}\,\frac{\phi\,(h)}{E_s} \quad \text{. . . . . (75)}$$

Since $Z_{max} = L/T$, this reduces to

$$A_y = \left[\frac{\phi\,(h)}{\phi\,(Z)}\,\frac{1}{Z_{max}}\right] a_y \quad \text{. . . . . (76)}$$

which is the relation shown in Fig. 14 on the horizontal axis of the graph of A_y versus Z. In a similar manner the relations shown on each of the other graphs have been obtained.

Actually, the rigid-pile is simply a special case of the elastic-pile theory in which the flexural stiffness E I is infinite. By a more general analysis, non-dimensional coefficients could have bee~ defined which would be common to both theories. The arrangement selected is justified on the basis that it is more convenient to use.

By the method of comparison of rigid-pile and elastic-pile results used in Fig. 14, the range of conditions under which each theory is applicable may be determined. The good agreement at $Z_{max} = 2$ indicates that the rigid-pile theory is a satisfactory substitute for the elastic-pile theory for all shorter piles and therefore should be used in this range to avoid the difficulties in difference-equation computations of very short stiff piles. Even at $Z_{max} = 3$, the rigid-pile theory provides a fairly good approximation.

APPLICATION OF GENERALIZED SOLUTIONS TO PILE DESIGN PROBLEMS

Non-dimensional curves for the form of soil modulus variation $E_s = k\,x$ have been available for some time for the elastic pile and have proved to be of value in the design of laterally loaded piles.[3] By use of techniques described

in this paper, the reader may recompute these non-dimensional curves or may derive new sets for any desired form of the soil modulus variation. This may be done for both the elastic and the rigid pile.

Solution of actual design problems for laterally loaded piles requires a trial and error adjustment of constants in the soil modulus functions, and perhaps in the form of the function. The steps of this procedure are as follows:

1. A soil modulus variation, usually $E_s = k\,x$, is selected and a trial value of k is assumed.
2. The value of Z_{max} is determined from elastic-pile theory. (If $Z_{max} < 2$, rigid-pile solutions should be used in the following steps).
3. Using non-dimensional coefficients, a trial deflection curve y (x) is computed. For the trials, deflections are needed only in the vicinity of Z less than about 1.0.
4. The computed trial values of deflection y are used as arguments to enter p - y relations which have been previously predicted for the given soil conditions. Revised values of $E_s = - p/y$ are obtained at each depth.
5. A revised value of the constant of soil modulus variation, and possibly a revised form of the soil modulus function itself, are determined by fitting to a plot of the revised E_s values.
6. A new trial is performed. The process is repeated until satisfactory compatibility is reached.

Satisfactory closure is indicated when the soil modulus values resulting from a trial are found to agree closely with the preceding set of values. The necessary degree of precision in soil modulus agreement can be determined by comparing the corresponding changes in values of maximum deflection and bending moment.

It seems reasonable to start the solution of any problem with an assumed soil modulus variation of the form $E_s = k\,x$. A more complex form should be used only after a reasonably good fit has been obtained by the use of $E_s = k\,x$, and then only if much doubt exists as to the correctness of approximating the computed soil modulus variations with the simple straight-line relation. In this connection, the comparisons given herein, among various forms of soil modulus functions, should be helpful as aids to judgment.

APPENDIX I. DIFFERENCE-EQUATION METHOD OF SOLUTION FOR ELASTIC-PILE THEORY

The basic differential equation of the elastic-pile theory, Eq. 25, may be rewritten in difference form as

$$E\,I\left[\frac{y_{i+2} - 4\,y_{i+1} + 6\,y_i - 4\,y_{i-1} + y_{i-2}}{(L/t)^4}\right] = -E_{s_i}\,y_i \quad \ldots\ldots (77)$$

Similarly, expressions may be written as follows for slope, S, moment, M, shear, V, and soil reaction, p.

$$S_i = \frac{1}{2(L/t)}\left(-y_{i+1} + y_{i-1}\right) \quad \ldots\ldots (78)$$

$$M_i = \frac{E\,I}{(L/t)^2}\left(y_{i+1} - 2\,y_i + y_{i-1}\right) \quad \ldots\ldots (79)$$

$$V_i = \frac{E\,I}{2(L/t)^3}\left(-y_{i+2} + 2\,y_{i+1} - 2\,y_{i-1} + y_{i-2}\right) \quad \ldots\ldots\ldots (80)$$

and

$$p_i = \frac{E\,I}{(L/t)^4}\left(y_{i+2} - 4\,y_{i+1} + 6\,y_i - 4\,y_{i-1} + y_{i-2}\right) \quad \ldots\ldots (81)$$

The subdivision of the pile into t increments is shown in Fig. 15, together with a summary of equations which are based on Eq. 77 and the Gleser method

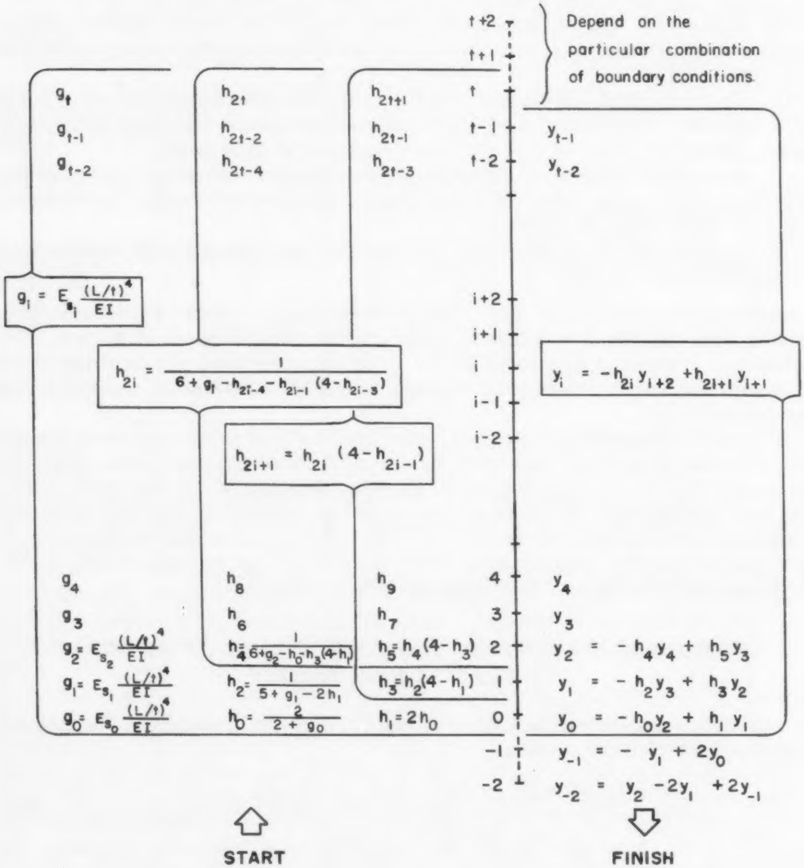

FIG. 15.—SUMMARY OF EQUATIONS AND COMPUTATION FORMAT

of once-through computation.[18] Two imaginary points at the top and two imaginary points at the bottom are added as a device for introducing the

18 "Lateral Load Tests on Vertical Fixed-Head and Free-Head Piles," by Sol M. Gleser, Symposium on Lateral Load Tests on Piles, ASTM Special Tech. Publ. No. 154, July, 1953, pp. 75-101.

boundary conditions. All necessary equations are given in Fig. 15 except those for y_t, y_{t+1}, and y_{t+2}. The format in Fig. 15 is for a constant E I, see Reese-Ginzbarg for an extension to accommodate step-tapered piles. When the known boundary conditions consist of a lateral load, P_t, and a moment, M_t, these equations are as follows:

$$y_t = \frac{j_4 h_{2t}(1 - h_{2t-2}) - j_3\left[h_{2t}(2 + 2 h_{2t-2} - h_{2t-2} h_{2t-3}) - h_{2t+1}\right]}{1 - h_{2t-2} - 2 h_{2t+1} + h_{2t-1} h_{2t+1} + h_{2t}(4 - 4 h_{2t-1} + 4 h_{2t-2} - h_{2t-4} + h_{2t-1} h_{2t-3} - 2 h_{2t-2} h_{2t-3} + h_{2t-2} h_{2t-4})} \quad \ldots (82)$$

$$y_{t+1} = \frac{j_3 + y_t(2 - h_{2t-1})}{1 - h_{2t-2}} \quad \ldots\ldots\ldots\ldots (83)$$

and

$$y_{t+2} = \frac{h_{2t+1}\, y_{t+1} - y_t}{h_{2t}} \quad \ldots\ldots\ldots\ldots (84)$$

The constants in Eqs. 82, 83, and 84 are as shown in Fig. 15 and by the following two definitions.

$$j_3 = \frac{(L/t)^2}{E\,I}\, M_t \quad \ldots\ldots\ldots\ldots (85)$$

and

$$j_4 = \frac{2(L/t)^3}{E\,I}\, P_t \quad \ldots\ldots\ldots\ldots (86)$$

Following are the steps necessary for computation.

1. Compute a value of g_i at each real point.
2. Proceeding from bottom to top, compute values of the two h-constants at each point.
3. Compute y_t, y_{t+1}, y_{t+2} from Eqs. 82, 83, and 84.
4. Proceeding from top to bottom, compute all other values of deflection y.
5. With y-values known, compute slopes, moments, and shears by Eqs. 78, 79, and 80. Rather than using Eq. 81 to compute soil reactions, it is more convenient and accurate to use the following expression which is based on Eq. 1.

$$p_i = - E_{s_i}\, y_i \quad \ldots\ldots\ldots\ldots (87)$$

APPENDIX II. NOTATION

The following symbols are adopted for use in the paper and for the guidance of discussers.

A_y, A_s, A_m, A_v, A_p = non-dimensional coefficients in elastic-pile theory, relating to an applied force P_t, for deflection, slope, moment, shear, and soil reaction respectively;

a_y, a_s, a_m, a_v, a_p = non-dimensional coefficients, same as A-coefficients, except for rigid-pile theory;

B_y, B_s, B_m, B_v, B_p = non-dimensional coefficients in elastic-pile theory, relating to an applied moment M_t, for deflection, slope, moment, shear, and soil reaction respectively;

b_y, b_s, b_m, b_v, b_p = non-dimensional coefficients, same as B-coefficients, except for rigid-pile theory;

E_s = soil modulus (force per unit length of pile and per unit deflection), in pounds per square inches;

$E I$ = flexural stiffness of pile, the product of modulus of elasticity and moment of inertia of pile cross section, in pound inches^2;

g = coefficient in the difference-equation solution;

h = coefficient in the difference-equation solution;

h = depth coefficient in rigid-pile theory, = x/L;

J = soil modulus constant, as defined for each form of $E_s(x)$, in pounds per square inch;

j = coefficient in the difference-equation solution;

k = constant of soil modulus variation in Eq. 52;

k_o, k_1, k_2 = constants of soil modulus variation in Eq. 53;

L = length of pile, in inches;

M = moment, in inch pounds;

M_t = moment at $x = 0$, in inch pounds;

n = exponent in Eq. 52 or Eq. 57;

P_t = shear at $x = 0$, in pounds;

p = soil reaction per unit of length of pile, in pounds per inch;

r_o, r_1, r_2 = constants in polynomial soil modulus functions;

S = slope;

T = relative stiffness factor, as defined for each $E_s(x)$, in inches;

t = number of increments in the difference-equation solution;

V = shear, in pounds;

x = depth below groundline, in inches;

y = lateral deflection, in inches;

Z = depth coefficient in elastic-pile theory;

Z_{max} = maximum value of elastic-pile theory depth coefficient L/T;

ϕ (h) = non-dimensional soil modulus function of rigid-pile theory, = E_s/J;

ϕ (Z) = non-dimensional soil modulus function of elastic-pile theory = $E_s\ T^4/(E\ I)$.

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DISCUSSION

ROCKFILL DAMS - DESIGN AND CONSTRUCTION PROBLEMS[a]

Closure by D. J. Bleifuss and James P. Hawke

D. J. BLEIFUSS,[5] F. ASCE AND JAMES P. HAWKE,[6] F. ASCE.—The writers are very appreciative of the comments of Mr. Steele. Mr. Steele has been responsible for the design and construction of many important dams, a number of which are rockfill dams of the type he designates as true rockfill dams (comprising a rockfill with an upstream face membrane of concrete, timber or gunite) and comments from his experience are most authorative.

Mr. Steele very properly emphasizes the importance of constructing rockfill dams so as to minimize the differential settlement throughout the dam. This becomes of particular importance in connection with many of the higher rockfill dams now (1960) under construction. Mr. Steele's comments regarding the 172-ft high Cogoti Dam in Chile, built with an average downstream slope 1.6 to 1 which withstood an extremely severe earthquake, is of special importance in our consideration of the earthquake resistance of high rockfill dams. The experience which Mr. Steele cites in connection with Sam Gabriel Dam No. 2, California and the La Joie Storage Dam, British Columbia, emphasizes the importance of his comments regarding care in selection of materials together with proper design and construction methods.

During the period which has elapsed since the writing of this paper, there has been a trend toward the design of increasingly higher rockfill dams. In the construction of some of these rockfill dams, it has been necessary to use available earth embankment and rockfill materials which, in some cases, were not ideal. In other instances rockfill dams have been constructed on foundations of granular materials which involved special problems in foundation stability affecting the design of the dam. Accompanying these factors has been the use of steeper outer slopes on the upstream and downstream shells of certain rockfill dams.

The comprehensive effect of the foregoing factors has been to make design problems more exacting. To adequately cope with these design factors we have an urgent need to know more about the actual shear strength of cohesionless material, both rockfill and gravel fill, under the high vertical and lateral pressures existing in dams over 400-ft high. Some laboratory work has been performed which would indicate that there may be a reduction in the shear strength of granual materials under such loading conditions.

The earth materials used in the core and the materials used in the filters become more critical for high dams. It is desirable to re-examine the problem

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[a] October, 1954, by D. J. Bleifuss and James P. Hawke.

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of construction pore pressures and consider carefully how closely our laboratory triaxial shear tests approximate the pore pressure conditions existing in the core of a high rockfill dam in the period prior to and during the initial filling of the reservoir.

From the standpoint of rigorous structural analysis our present methods of stability analysis are open to question. The assumption of failure along a thin slice of unity width introduces approximations, particularly in the case of steeply sloping abutments which may be overly conservative. This would indicate a need for more realistic methods of stability analysis based on the various modes of failures which might rationally be expected.

Certain high rockfill dams involve diversion and closure problems which require the closure section of the dam be placed in a single dry season. This leads to a need to make the end slopes of the initial stage of the embankment as steep as practicable in order to minimize the amount of material to be placed in the closure section. Studies of such end slopes by the method of thin slices leads to unrealistically flat slopes since the unit width section may lie entirely within the impervious core. Obviously, however, any large scale failure of the end slope would involve the adjacent cohesionless shells, and the effects of the shells should be considered in the stability analysis. This is being done on recent projects with which the writer is connected and it is hoped that further study along this line may lead to more realistic and satisfactory methods of stability analysis of composite earth and rockfill dams.

A REVIEW OF THE ENGINEERING CHARACTERISTICS OF PEAT[a]

Closure by Ivan C. MacFarlane

IVAN C. MAC FARLANE.[1]—The writer is appreciative of the constructive discussion given to his paper by Messrs. Kenney, Zegarra, Hanrahan, and Keene. The examples noted of personal investigations of peat are of particular interest and value since they add to the sum total of the knowledge on the subject of engineering properties of peat. The writer agrees with Mr. Hanrahan's comment that peat in many respects is quite different from other materials studied in soil mechanics. It is also true that even routine laboratory procedures require modifications when testing peat. The application of theories and techniques of soil mechanics, or modifications thereof, has however proved to be a useful approach to the understanding of the mechanical properties of peat.

In suggesting terminology for peat properties, the writer recognizes that he was placing himself in a vulnerable position. He is the first to agree that the terms and definitions probably will not be accepted by everyone, but he hopes that they will be generally acceptable to most workers in the field. He concurs with Mr. Zegarra's contention that the terms "wet density" and "dry density" are better expressed by the recommended American Society of Civil Engineers (ASCE) definitions for "wet unit weight" and "dry unit weight" respectively. It is further agreed that the American Society of Civil Engineers (ASCE) definition for "water holding capacity" (the smallest value to which water content of a soil can be reduced by gravity drainage) is preferable to the writer's definition (the ability of the soil to take up and hold water).

Percentage ash (or organic content) is determined by igniting the peat in a muffle (electrical) furnace at 800°C (1472°F) for 1 hr to 2 hr, depending on the type of peat. This is in accordance with the procedure of various other investigators but the ignition temperature is only half that used by Mr. Keene. Mr. Zegarra referred to the difficulty of obtaining a good representative sample of peat for laboratory testing, a difficulty common to all who have attempted such testing for peat. An investigation of this particular aspect of the peat problem is being performed by the Engineering Department of the University of Alberta, Canada. It is to be hoped that if Mr. Zegarra continued his studies on peat as he has indicated, the results will be published.

Since the author's original paper was written, several additional references to engineering aspects of peat have come to light.

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COMPRESSIBILITY AS THE BASIS FOR SOIL BEARING VALUE[a]

Closure by B. K. Hough

B. K. HOUGH,[1] F. ASCE.—A sufficient number of discussions was submitted to reflect a wide range of opinion on the subject of the paper. It was the writer's hope that this development would take place. Despite the divergence in the viewpoints of the various writers, certain generalities are possible and will be presented before commenting in detail on each discussion.

It is a contention of the writer that design of foundations chiefly on the basis of rupture theory is unsatisfactory since this gives no assurance of equal settlement under the design loads. The writer is of the opinion that major emphasis has nevertheless been placed on establishing the safe or allowable soil bearing capacity as an arbitrary fraction (1/2 or 1/3) of the ultimate and that consideration of settlement has come to be regarded as a matter of secondary importance. The writer therefore proposed certain new procedures for determining soil bearing values for use in proportioning spread foundations.

Although a majority of the discussors appear to share in whole or in major part the writer's estimate of the present situation, a good many have expressed disagreement. In most cases, those who disagree feel that the importance of settlement considerations has been fully recognized and adequately covered in the literature and in present-day courses of instruction.

References to the standard texts may appear to support the position of the minority. However, it will be found that in many cases, such discussions of settlement considerations as are presented in these texts are preceded by an extensive development of rupture theory and advocacy of the criterion of a constant safety factor. The order of presentation and relative emphasis on the two subjects is such that it is these very publications which have contributed in many cases to the impression that rupture theory is the controlling factor. To put it in another way, it was not until these texts and other publications in the field of soils engineering began to appear, that the idea of providing a constant factor of safety against rupture became a recognized basis of foundation design.

In this controversy, however, there is a much more serious consideration. Those who claim that the importance of settlement analysis has already been fully recognized, do so with at least an implication that the methods which are currently advocated for settlement analysis are entirely satisfactory. There is, in some cases, almost a suggestion that the subject is closed. With respect at least to the treatment of settlement of foundations on sand, this position can be attacked. In the existing literature it appears to have been accepted that even under moderate loading, the settlement of footings on sand is chiefly associated with lateral strain and hence is directly related to surcharge loading. It is the

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[a] August, 1959, by B. K. Hough.

[1] Consulting Engr., Ithaca, N. Y.; Prof. of Civ. Engrg., Lehigh Univ., Bethlehem, Pa.

writer's thesis, on the contrary, that under moderate loading the settlement of spread foundations on soils of every type is due chiefly to one-dimensional soil compression. Thus, insofar as there is any merit whatever in the writer's arguments, it can scarcely be claimed that previously published discussions of settlement have exhausted the subject. In later sections of this closure, further evidence in support of the writer's views is presented. Detailed comments on each discussion are now submitted:

Mr. Praszker contends that the equations developed by the writer should be used only with the greatest care and only by those thoroughly familiar with the theory of consolidation. If there is an implication here that there is more need for care in using the writer's equations than those of others, it is unsupported. All mathematical processes require great care and understanding in application. Mr. Praszker's further admonition that not only care but familiarity with the theory of consolidation is required is difficult to accept since the writer's paper deals with soil compressibility, not the theory of consolidation. Since Mr. Praszker seems to have confused compressibility with consolidation, not only in his opening remarks, but generally throughout his discussion, the following clarification is given:

When subjected to compression, most (if not all) soils, regardless of particle size, texture, gradation, plasticity, or degree of saturation, under go a decrease in volume, in short, compression. It has become established simply through observation that compression diagrams representing one-dimensional compression have certain characteristics, including an approximately linear shape for short sections of the virgin curve on a semi-log plot. Compressibility theories exist,[2] but none was used in establishing this characteristic of compression diagrams. Compression diagrams for the fine textured, saturated materials to which Mr. Praszker refers are known to be the same in this respect (that is, shape) as diagrams for the granular, semi-saturated materials which he mentions by way of contrast.

Consolidation theory provides a relationship between the hydrostatic excess pore pressure in a saturated soil mass and the elapsed time after application of load increment, and thus serves as the basis for estimating the rate (not total amount) of compression. The theory is limited by basic assumption to saturated soil and by practical considerations to fine grained, saturated soil but it is of little value even for its intended purpose in analysis of a total settlement which in any case is to be restricted to the order of magnitude of 3/4 in. or less. In contrast, no analysis of settlement whatever can be made without information on soil compressibility. Mr. Praszker's contention that the compression index C_c applies only to "fine textured, saturated materials which will undergo consolidation by virtue of gradually giving up the water from their voids upon the application of pressure increment" is unwarranted and unsupportable.

Mr. Praszker notes that the writer does not explain how the value of C_c can be obtained and interpreted for dry or semi-saturated sands and states that it would be of interest to know how the index was obtained for sand and gravel. For sands, the compression index can be obtained without difficulty from results of conventional compression tests. This approach is certainly not unprecedented. D. W. Taylor,[3] for example, devotes a section of his text to "Com-

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[2] "Basic Soils Engineering," by B. K. Hough, Ronald Press Co., New York, N. Y., 1957, pp. 98-100.

[3] "Fundamentals of Soil Mechanics," by D. W. Taylor, John Wiley & Sons, Inc., 1948, pp. 215-217.

pression of Sands." Terzaghi and Peck[4] give a discussion of "Compressibility of Crushed Minerals (including sand) and Remolded Soils" and present typical compression curves for loose and dense sand and mixtures of sand and mica. An analytical procedure for determining the compression index of soils ranging from coarse-grained sands and gravels to inorganic silts and clays has been developed by the writer.[5] Mr. Praszker, in raising this question, leaves the impression that he considers settlement analysis of interest only in fine grained, saturated soils. In engineering practice, however, the compressibility of sand and other granular soils, whether saturated or not, is often a vital factor, for example, in choosing between spread foundations and piling, a choice which may involve a cost of hundreds of thousands of dollars.

To Mr. Praszker's charge that the writer has failed to define the term Δp, rebuttal can be made on several grounds. In the first place, if a term such as p is adequately defined, the elementary term Δp has an accepted meaning in any field which scarcely requires explanation. Secondly, the meaning of the term, even though it was not given in words, is clearly illustrated in Fig. 2. Thirdly, he will find the definition of the term Δp given together with the definition of the term p_i immediately following Eq. 10. In this same area, one might question the need for the definitions of other elementary terms which Mr. Praszker undertakes to furnish in his discussion.

Mr. Praszker notes that all expressions developed in the paper relate solely to one case and one case only, namely when the "consolidable" layer directly underlies the footing. This statement and those immediately following it which are illustrated by Mr. Praszker's Diagram 1, Figs. 1 through 3, are intended to draw attention to the possibility that in some cases there may be some stratification of the bearing materials including relatively weak layers within the zone of significant stress. The possibility of such stratification most certainly exists, but when it is recognized that the zone of significant stress is usually of the same order of magnitude as the footing width, it is possible to show that stratification within this relatively small depth is (a) more apt to be the exception than the rule and (b) that when and if it does occur, many simple, practical methods of dealing with it are available. Mr. Praszker's Figs. 1 and 2 serve merely to illustrate the stratified conditions to which he refers. The situation represented by Mr. Praszker's Fig. 3 represents a large mat foundation integral with the foundation walls, rather than any type of individual spread footing. It therefore bears no relation to any situation discussed by the writer and the reason for its inclusion is not apparent. Furthermore, it may be observed that the writer's entire development of analytical methods is presented under the heading "Compression of Unstratified Soil." The possible occurrence of a relatively thin substratum of compressible material is acknowledged but excluded from the scope of the discussion. The point is made by the writer and is here reiterated, that it is the unstratified case which has heretofor been the difficult problem analytically, since there is no physical limitation on the zone in which the major compression will occur.

The writer cannot accept Mr. Praszker's contention that p_i (the initial body stress) "need not at all be the initial stress due either to the pre-excavation surcharge or the existing surcharge." The initial stress at a point as the term is used by the writer is the stress existing initially, that is, at the time that

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4 "Soil Mechanics in Engineering Practice," by K. Terzaghi and R. Peck, John Wiley & Sons, Inc., 1948, pp. 57-61.

5 "Basic Soils Engineering," by B. K. Hough, Ronald Press Co., 1957, pp. 113-115.

some proposed construction is initiated. This stress (vertical, compressive stress is implied) is simply a function of depth and unit weight of soil (Eq. 9) whether the soil is normally loaded, preconsolidated, compacted, or extra-sensitive. (The soil may perhaps not be fully consolidated under p_i but this is outside the scope of this discussion). The past history of loading and the physical characteristics of the soil are reflected in the value of C_c which is chosen for use in a particular application. If the initial stress for a preconsolidated soil were taken to be much higher than the stress due to the weight of the existing overburden, as Mr. Praszker suggests, there would be a lack of static equilibrium in the soil mass which would result in a bodily upheaval of the entire formation.

Mr. Praszker's concluding remarks appear to convey general disapproval of the use of "direct and reciprocal" algebraic expressions in a technical publication and in respect to this sentiment, the writer does not choose to comment.

Mr. Grosswirth's general concurrence in the writer's conclusions is most encouraging. With reference to the three exceptions which are noted, the following comments may be made. Under his point No. 1, Mr. Grosswirth suggests that in Fig. 8 (a) and related statements, a distinction might be made between normally loaded and precompressed clays. His argument is, that with contact pressures limited only by considerations of rupture, settlement (that is, total settlement) might well be excessive for the former but not for the latter. He states in effect that if the curves of Fig. 8 (a) represent normally loaded clay, the curves for a precompressed clay might all plot above the $\left(p_{ult}/S\ F\right)$-line. However, if the size of the smallest footing is selected as has been recommended, to provide a given factor of safety against rupture and the sizes of all other footings are made such, as to provide for equal settlement, design curves will all plot below the $\left(p_{ult}/S\ F\right)$-line even for precompressed clays. The writer is concerned with the development of procedures which can be applied to eliminate or minimize differential settlements and for the sake of consistency would prefer to use such procedures in all cases, even when the total anticipated settlement is very small. In his second point, Mr. Grosswirth brings into the discussion, as have other writers, the possible effect of lateral strain on the allowable bearing capacity of soil beneath footings with and without surcharge, and questions the writer's conclusion that "the total load on surcharged footings (for a given settlement) should be less than on similar footings without surcharge." The referenced conclusion relates to the action of footings under moderate loading only for which condition it has been assumed that settlement is due chiefly to one-dimensional compression. For this assumption the writer's development is complete and rigorous and the conclusions as to surcharge effect need no revision. For a discussion of the probable order of magnitude of effects due to lateral strain Mr. Grosswirth is referred to the writer's subsequent discussion of Mr. Peck's comments.

Mr. Grosswirth's third point is that the writer's proposal to vary contact pressures inversely with footing size is in the nature of a refinement which may be inconsistent with the accuracy to which soil conditions can be determined under practical conditions. It is readily conceded that, in almost all cases, our knowledge of soil conditions is and will be incomplete. However, the writer will not agree that this should limit our development of basic theory. It is believed safe to say that present methods for analyzing every type of problem in soils engineering involve the assumption that within certain distances, at least, the soil is realtively uniform in some respect, as to permeability, for example, or

shearing strangth, density, or compressibility. There will always be many cases where the only possible approximation of an analysis of soil bearing capacity is to establish presumptive bearing values, but this should not deter us from attempting to develop more rational methods for use under suitable conditions.

The discussion by Mr. Peck was especially welcomed by the writer since it serves to identify Mr. Peck with some of the views which the writer has called into question and thus sets the stage for direct debate.

In his discussion, Mr. Peck states categorically that certain of the writer's conclusions are incorrect, certain assumptions unsuitable, and certain suggested analytical procedures unacceptable. The following evidence to the contrary is presented:

Mr. Peck is among those who feel that in present texts, the importance of settlement analysis has not been neglected or over-shadowed by emphasis on rupture theory and also that the present coverage of the subject is adequate. However, Mr. Peck is an advocate (possibly an originator) of the theory that even under moderate loading, the settlement of footings on sand is chiefly a function of the extent to which lateral strains can occur in the bearing materials, and that the opportunity for lateral strain is dependent on the confining pressure, and hence on surcharge. The possibility of direct compression of the sand appears to be virtually excluded in the discussions which appear in his publications.[6,7] It may be noted that with clay soils, these assumptions are reversed. Referring principally to the case of a layer of clay with sand above and below, it is postulated by Mr. Peck that the tendency for lateral strains to occur in clay is almost completely overcome by adhesion between the clay and the stiffer materials at its faces. On this premise, the settlement of foundations underlain by clay (layers) is attributed chiefly to one-dimensional compression of the clay, rather than lateral strain.

From the foregoing it can be seen that it is chiefly in respect to the mechanics of the settlement of footings on sand that the writer and Mr. Peck are in disagreement. Pending the development of a rigorous theoretical analysis of the deformation of particulate materials under distributed surface loading of limited lateral extent, conflicting opinions on this subject can best be tested pragmatically. For this purpose, results obtained by use of analytical procedures recommended by the writer may be compared with available load-settlement data. Such a comparison should have two objectives. One is to show that settlement curves based on the writer's analytical procedures are similar in shape to the initial sections on conventional curves. The other is to show that the order of magnitude of the settlement indicated by the recommended procedures is the same as that commonly observed in practice.

Before proceeding with such a comparison, one incidental matter requires consideration. As the writer uses the term, compressibility denotes the extent to which a soil will compress or decrease in volume under a given loading in a completely confined condition, as in a confined compression test. It would appear, however, that Mr. Peck has a different concept of the meaning of the term. In his discussion, Mr. Peck states "if a sand is surcharged, its compressibility decreases markedly as a consequence of the increase in all-around pressure." The writer will readily accept the assertion that under a given loading a greater

6 "Soil Mechanics in Engineering Practice," by K. Terzaghi and R. Peck, John Wiley & Sons, Inc., 1948.

7 "Foundation Engineering," by Peck, Hanson, and Thornburn, John Wiley & Sons, Inc., 1953.

vertical deformation will occur in an unconfined soil than in a soil subjected to confining pressure or rigid lateral restraint. However, it is not considered that this is due to a difference in compressibility of the soil in the two cases. It is simply that compression alone occurs in the one case; both compression and lateral strain in the other.

Considering, then, only the direct compressions which would occur if the bearing material were rigidly confined laterally, the settlement to be expected in various conditions of practical interest would be as represented in Fig. 11. The diagrams of Fig. 11 are based on data obtained from Fig. 5 of the writer's paper. The three sets of curves distinguished by full lines, dotted lines, and dashed lines are for soils with bearing capacity index values of 50, 100, and 150, respectively. Referring to Fig. 3 of the paper, it can be seen that not only sand, but all the common soil types are thus represented. The curves in each

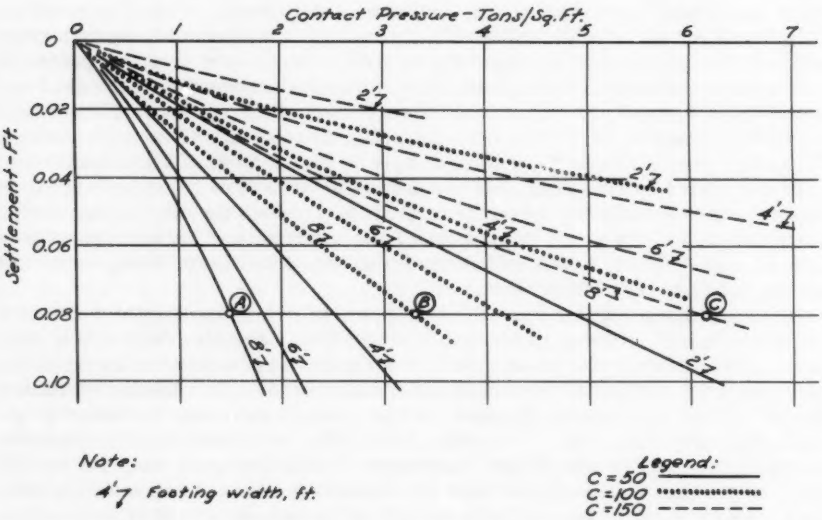

FIG. 11

set are for footings having widths of 2 ft, 4 ft, 6 ft, and 8 ft, respectively as noted.

As to shape, some of the curves of Fig. 11 are in agreement with the sections of conventional curves representing settlement under moderate loading in that they are either straight or have a slight downward curvature. Others, however, have a slight upward curvature. To bring these latter curves down to a linear configuration, their ordinates at successive points should be increased. This increase would not be very large on a percentage basis, however. If the contributions of direct compression and lateral strain to the total settlement in an actual case were in proportion respectively to the ordinates of these curves and the increment necessary to convert the curves to straight lines, it certainly could not be claimed that lateral strain was the major factor and direct compression negligible.

Further examination shows that the more compressible the soil (for example, the lower the bearing capacity index value) the more nearly the diagrams of Fig. 11 correspond in shape to actual settlement diagrams. This suggests that with relatively compressible materials, whether they be loose sand or soft to medium clay, lateral strains may well be negligible under moderate loading. None of the curves suggests that settlement is chiefly due to lateral strain.

As to magnitude of settlement, reference may be made to material presented in Mr. Peck's publications,[6,7] in which it is stated that the known facts regarding settlement of footings on sand can be represented by a set of curves in a single diagram relating the soil pressure for a settlement of 1 in. to footing width and soil density. It may be noted that in this representation there is no identification of the basic data but surprisingly, no reference to confining pressure or surcharge. The reason for terming this omission "surprising" is that in his discussion of the writer's paper, Mr. Peck indicates that even under moderate loading, surcharge is such an important factor that certain of the writer's conclusions would be either correct or "invalid or at least too sweeping," depending on whether column loads are applied before or after surcharging. Overlooking this apparent inconsistency, however, the following comparisons can be made:

The full-line curves in Fig. 11 represent conditions for a well graded fine to medium silty sand with a penetration resistance value N of about 15 blows per ft. Point A represents a condition of 1 in. settlement of a footing with 8-ft width on such material; an 8-ft width is selected as the basis of comparison because footings of greater width are uncommon despite the inclusion in Mr. Peck's figure of widths up to 20 ft. It will be noted that Fig. 11 indicates that a contact pressure of approximately $1\frac{1}{2}$ tons per sq ft is required to produce such a settlement under the basic assumption that only direct compression occurs. For the same loading on the same or similar soil, Mr. Peck's curves indicate that a contact pressure of about $1\frac{3}{8}$ tons per sq ft is required for a 1 in. settlement.

The dotted line curves in Fig. 11 represent conditions for the same sand with a penetration resistance value of about 40 blows per ft. Point B indicates that a pressure of about 3.3 tons per sq ft is required to cause an 8 ft footing to settle 1 in. Mr. Peck's curves for the same conditions indicate that a pressure of slightly over 4 tons per sq ft is required.

The dashed line curves in Fig. 11 represent a sand with an N-value of approximately 60. Point C indicates that on such material a footing 8 ft in width would settle 1 in. under a pressure of a little over 6 tons per sq ft. For these conditions, Mr. Peck's curves give almost exactly the same result.

Thus it can be seen that all the settlement which Mr. Peck predicts, at least for the foregoing conditions, can be accounted for by direct compression alone. If Mr. Peck's curves are intended to represent settlement due to lateral strain only, they would appear to be in error by a factor of about 100%, because the indicated compression would occur in addition to lateral strain in each case.

Returning now to statements made in Mr. Peck's discussions, reference is made to the assertion that some of the writer's "incorrect" conclusions are due to the "unsuitable" assumption that all types of soil have similar stress-deformation characteristics. If it can be shown that under moderate loading the chief consideration with all types of soil is direct compression, then it is not so illogical to assume that the particular stress-deformation data of practical value is the data represented by a field compression diagram (representing one-dimensional, vertical compression only). With certain exceptions, these diagrams for sand and clay alike have one common characteristic, namely, that

their shape on a semi-logarithmic plot within the usual range of practical interest is essentially the same—linear. Thus to the extent noted previously, the stress-deformation characteristics which are of interest in proportioning structural foundations for equal settlement are the same for all types of soil since in each case they may be represented by the compression index of the soil. The writer is aware of distinctions which must sometimes be made in the case of extra-sensitive soils and precompressed clays but submits that the existence of certain exceptions does not invalidate the basic premise.

Reviewing Mr. Peck's discussion, it can now be seen that the somewhat unfavorable comments contained therein are limited in application chiefly to the particular case of footings on sand, and with respect to this particular case, Mr. Peck's position is not as impregnable as it might at first appear. Specifically, his position is based almost entirely on his contention that even under moderate loading, the settlement of foundations on sand is due chiefly to lateral strain in the sand, and thus is directly related to surcharge. The writer feels that sufficient evidence to the contrary has been presented herein, if not to disprove this theory entirely, at least to justify a very serious review and reevaluation of present beliefs on the subject, widely accepted though they may be. Within the basic assumptions made by the writer, the development of the main thesis of the paper will be found to be rigorous; the conclusions logical and fully documented. Mr. Peck has pointed out no flaw in the development. In particular, the conclusion that for equal settlement under moderate loading the total column load on a surcharged footing should be less than on a similar footing without surcharge, will be found to be fully supported both mathematically and in a qualitative sense, it being clear that the surcharge at the sides of the footing contributes to the compressive stress beneath the footing.

The subject of bearing-value determination deserves continuing study. A complete theoretical analysis of the deformation of the bearing materials under moderate loading, for example, would be particularly helpful. Although the writer has so far concerned himself only with the effects of direct compression, there has been no denial of the supposition that lateral strains must contribute to settlement in some degree. It is to be hopes that Mr. Peck can be interested in further investigation and research in this field, and that with his contributions, significant progress will be made.

Mr. Lawson's general agreement with the writer's assumptions and conclusions is noted. In reply to the question raised in Mr. Lawson's discussion about use of electronic computers, it can be stated that as yet such computers have not been used by the writer. However it appears that Messrs. Brown and Crawford and also Mr. Kellogg and Mr. Murphree have used computers in solving some of the writer's equations. Since their remain many other conditions to be analyzed besides those which have thus far been considered, Mr. Lawson's suggestion is certainly a timely one.

The discussion by Messrs. Brown and Crawford is encouraging in that it presents evidence from independent investigations to indicate that, with certain exceptions, the curves of Fig. 3 of the paper give values of approximately the correct order of magnitude. The following information on the origin and development of these curves is supplied in response to the request of the discussors:

The writer has for some years been of the opinion that if the maximum and minimum void ratios of a soil are known at least approximately, and if the in-place density of the soil can be determined as by correlation with spoon blows, the compression index of the soil or the slope of its field compression diagram

can be estimated with reasonable accuracy without resort to compression tests. This subject and methods for accomplishing the indicated result are discussed in detail in the author's text.[8] The curves of Fig. 3 of the paper were constructed by making a series of computations by the methods outlined in the referenced text. For example, for a given material such as "clean, uniform, medium sand," assumptions were first made as to the maximum and minimum void ratios of this material. In this connection, reference was made to Fig. 2-9, p. 30-31 of the text. The in-place density or void ratio of the material at given values of standard penetration resistance was then calculated by reference to the correlation between spoon blows and relative density which is presented in Fig. 12-3, p. 357 of the text. Compression index values were then estimated as noted previously and the e and C_c values determined in this manner were then used to calculate bearing capacity index values.

Much can be done to improve the reliability of the values obtained in this way, and the writer is particularly interested in the discussors' comment that the present discrepancies appear to be grouped, and their suggestion that better agreement might be obtained by adjustment of some of the curves. This seems entirely reasonable and the writer will hope to hear further reports from the discussors which might indicate the extent to which such adjustment should be made.

In response to the discussors' request for a brief discussion of the usual limits of linearity of the semi-log compression diagram, the writer can say that this subject has also been covered in some detail in his text.[9] Opinions on this subject are bound to differ somewhat since there is, at present, no completely developed and accepted compressibility theory. Conclusions can only be based on observation of numerous test curves, each of which is subject in some degree to experimental errors. What is worse, there is practically no empirical information available on the relationship between laboratory test curves and field compression diagrams. However, the writer has found over a period of many years' experience that reasonably good correlation between predicted and observed settlements can be obtained if it is assumed that within the range of ordinary loading the field compression diagram is essentially linear on a semi-logarithmic plot. It is to be especially emphasized that this is true for all types of soil. The only difference between a sand and a clay in this respect is the difference in the slope (not the shape) of the curve over a given pressure range.

Since certain of the comments of Mr. Ireland parallel those of Mr. Peck, the writer will comment herein only on points raised independently by the former. One such point is the question as to the value of C_c intended by the writer for use in his proposed term, bearing capacity index. The direct answer to this question is that in this connection and generally throughout his paper, the writer intends that a value representing the slope of the semi-log field compression diagram between the points e_1 (initial void ratio) and e_2 (final void ratio) will be used wherever the term C_c is written.

It is acknowledged that this usage is not in complete agreement with the present conventions in all respects, and, therefore, it is well that Mr. Ireland has raised this point. Strictly speaking, the term C_c is reserved for application only to the slope of the so-called virgin compression curve for a laboratory test specimen. Other terms are used for other branches of the compression diagrams

8 "Basic Soils Engineering," by B. K. Hough, Ronald Press Co., New York, N. Y., 1957, pp. 113-120.

9 Ibid., p. 104-112.

for laboratory text specimens. In particular, it is acknowledged that distinction is and should be made between the slope of a curve representing recompression after reduction of the test loading and the slope of the virgin compression curve. On this basis, then, it is held that a difference in the slope of the compression diagram for a precompressed soil is to be expected in pressure ranges below and above the point of maximum previous compression. As long as the field compression diagram is reasonably linear within the range of expected loading, however, and as long as the correct C_c value is used for each range, such distinctions are chiefly of academic interest. Thus the writer has chosen to use the same term, namely, compression index, in all cases to denote the slope of linear sections of the compression diagram. It is accepted that there will be marked differences in the slopes of the diagrams and compression index values for the various classes of soil to which Mr. Ireland refers.

In respect to Mr. Ireland's comments on the difficulty of making accurate determinations of C_c, the writer is in general agreement. However, it is not claimed by the writer that accurate C_c values are required for the purpose of eliminating differential settlement which is the main objective of procedures proposed in the paper. If the C_c value used for this purpose is somewhat different from the true value, the only consequence is that all the footings will settle somewhat more or less than predicted. The important consideration is that they will all settle approximately the same amount. Since highly accurate determinations of C_c are not essential in this connection, the approximations proposed by the writer are to a considerable extent justifiable.

Mr. Ireland concludes by stating that to him "it seems illogical to attempt to proportion individual footings based on precise settlement computations." Great accuracy in settlement analysis may be difficult to obtain under certain conditions, but the concept of eliminating differential settlements can scarcely be termed illogical. As a design criterion, what could be more logical? The difficulties do not lessen the merits of the basic concept.

Mr. Koo's contributions are acknowledged with sincere appreciation. The cases for which he has supplied additional equations are definitely of practical interest. Eventual development of complete design procedures will require inclusion of the three cases which Mr. Koo has analyzed as well as a number of others in which these equations can be utilized in varying degree.

Mr. Scheer has requested the writer to defend his thesis that the settlement of a surface footing resting directly on the compressible stratum is due to one-dimensional consolidation. Most of the material necessary for such a defense has been presented previously, chiefly in the material concerning Mr. Peck's discussion. The subject is also mentioned in the discussion of Mr. Chinn's contribution. A few additional comments may perhaps be made:

In the first place, Mr. Scheer is apparently under the misconception that the writer advocates use of the one-dimensional consolidation theory for estimating the settlement of footings. As noted in remarks concerning Mr. Praszker, consolidation theory is of no value whatever in estimating the amount of settlement to be expected under a given loading. For this latter purpose it is necessary to have data on soil compressibility such as that obtainable from a confined compression test. The pressure-void ratio relations established in this way are experimental or empirical in nature, not theoretical. The writer's paper proposes use of soil compressibility as the chief factor in controlling or estimating settlement. There is no reference to the theory of consolidation.

It may also be noted that the writer's paper is not chiefly devoted to the case of the surface footing. It is stated in the paper that "Equation (14) applies to

the special and rather unusual case of a surface loading and has been developed chiefly to give an indication of the type of expression required for the purpose under discussion. Structural foundations are almost invariably constructed at some depth."

Mr. Scheer is correct, however, in his impression that the writer is of the opinion that under moderate loading, the settlement of structural foundations is due chiefly to one-dimensional soil compression. The writer will make no effort to represent that surface footings or unsurcharged footings at any level resting directly on the compressible stratum are exceptions. The definition of "moderate loading" may perhaps require reconsideration if such a statement is to be made. Possibly it would be well to say that "moderate loading" is loading which does not exceed approximately 50% of the ultimate and which does not exceed the average code values.

In saying that settlement is due chiefly to compression, the writer does not intend to exclude the possibility that some fraction of the settlement may be due to lateral strain in the bearing materials. In so far as lateral strain contributes to settlement, it seems axiomatic that an unsurcharged footing will experience more settlement under a given loading than one which is surcharged. It is the writer's theory, however, that in either case, one-dimensional soil compression is the major factor.

Evidence that this theory is valid or, at the very least, cannot be lightly dismissed, is found in the fact that the amount of settlement of unsurcharged footings due to one-dimensional compression only, proves to be almost exactly as much as that predicted by advocates of the theory that lateral strain is the chief consideration. This evidence will be found in the writer's comments on the discussion by Mr. Peck. The writer can also report that for many years he has been estimating settlement under design loading on the basis of one-dimensional, primary compression only and zero lateral strain. There has as yet been no instance of observed settlement exceeding the predicted. In fact, even with the foregoing assumptions, the usual experience is that the observed settlements are considerably less than the predicted.

In the face of such evidence, it is suggested that rather than questioning the writer's theory on the ground that it is not in accord with the opinions of a number of prominent authors, Mr. Scheer might ask the referenced authors to defend their theories.

The writer notes Mr. Siecke's general concurrence with his views and the value of the recommended procedures. The need which Mr. Siecke cites for analytical methods suitable for use when very detailed, costly programs of sampling and laboratory testing are impractical, is one which the writer has long recognized. The development of the writer's paper stems, in fact, very largely from this recognition.

Mr. Greer's general endorsement of the writer's thesis that compressibility is the logical index of bearing capacity is welcomed. The writer surmises, however, that Mr. Greer may be unduly concerned as to the accuracy with which it is possible to determine such soil constants as the proposed bearing capacity index. As noted in the writer's comments on Mr. Ireland's discussion, exact estimates of total settlement are not required to realize the objective of eliminating excessive differential settlements. The writer believes that with even a rough approximation of the true index value, total settlements can be held to tolerable values and differential settlements can be reduced to negligible proportions. On this premise, the writer feels confident that bearing capacity index values satisfactory for the stated purposes can be estimated from such

readily available information as textural classification and in-place density without recourse to data from compression tests on undisturbed soil specimens.

The writer is somewhat puzzled by Mr. Greer's statement that laboratory determination of the bearing capacity index of (cohesionless) soils is not feasible at the present state of the science (1960). Determination of the bearing capacity index requires only evaluation of the in-place void ratio and compression index of the soil. Where a high degree of accuracy is desired, both values can be determined from standard tests on undisturbed samples. Such samples can be obtained with relatively little difficulty by manual sampling operations in test pits; with greater difficulty, from borings. However, reasonable approximations of these values can be obtained from existing correlations with standard penetration resistance values, whether samples are obtained or not.

Mr. Harlan states that establishment of allowable bearing values and design of foundations are functions of the foundation engineer. He expresses doubt that the usual practice is to resort to approximate or empirical procedures. To this it may be observed that an overwhelming majority of the structural foundations constructed today are not designed by foundation engineers. On the basis of some 30 yr experience and a very active and varied present practice, the writer would venture the opinion that less than 10% of the foundations constructed annually are even reviewed let alone designed by engineers specializing in soils and foundation engineering. Design practices in general are, therefore, approximately as described in the writer's paper.

Mr. Harlan's comments on the value and proposed use of the term "bearing capacity index" which is one of the features of the writer's development, appear to be based on a number of misconceptions. Objection to the term is raised among other things because, since it involves both the compression index and the initial void ratio, "few designers will grasp its significance and limitations." In this respect, the term is not in the least degree different from the term "compression ratio" which has been in the literature for many years without arousing any serious protests. To be sure, few of those who actually design footings at the present time are aware of the existence of either term, but if design of foundations is to be considered an important function of the foundation engineer, as Mr. Harlan proposes, rather than a chore to be performed by a structural draftsman, then the fact that a certain index value is a function of two soil constants should not in itself be a serious objection. When it is realized that even structural draftsmen have learned to utilize intelligently such terms as moment of inertia and section modulus, there should be even less objection.

One of the misconceptions which the writer attributes to Mr. Harlan is the latter's assumption that the theoretical development of the paper is limited in application to normally consolidated soil deposits; this is simply not true. Neither is it true, incidentally, that the writer is unaware of the distinction between normally loaded and precompressed soil formations or of the importance of the geology and stress history of the foundation materials. The only condition which must be met in order to apply the equations of the paper is that for the bearing material under consideration, the field compression diagram when constructed as a semi-logarithmic plot be linear in shape across the expected range of pressure variation. With precompressed formations there is the chance that the expected range of pressure variation might possibly extend beyond the maximum previous compressive stress. It is true as Mr. Harlan points out, that if this occurred at all, it would be most likely in the case of the smallest of a group of footings. It would admittedly be necessary to assume that the designer would be cognizant of and alert to this possibility. But the writer is at a loss to under-

stand why the procedures recommended in this paper should be considered any more vulnerable or subject to criticizm in this respect than any other procedure. Under any procedure, the design of one of a group of footings for a pressure application extending beyond the precompression point would definitely be a special task. Except for this one inherently special and quite unlikely situation, it is only necessary in using the procedures recommended by the writer to determine the proper bearing capacity index for the soil in question, be it normally loaded or precompressed, and then to utilize this value in the recommended manner.

Perhaps the chief misconception revealed by Mr. Harlan, however, as well as by a number of others, is that the writer has proposed that bearing capacity index values be determined solely by correlation with penetration resistance values. It is on the assumption that this has in fact been proposed that Mr. Harlan and others have drawn the conclusion that since the correlation of penetration resistance with any soil index value can be no more than a rough approximation at best, the whole procedure which the writer has proposed is a rough approximation at best. In contrast to this misconception it should be realized that the bearing capacity index can be determined in any given case with any degree of accuracy desired. The compression index and the in-place void ratio can be determined separately and their determination can be based on tests on as many undisturbed samples as anyone chooses to conduct. For those who are in a position to command a testing program with each investigation, it would be deplorable to depend in any way on penetration resistance values except perhaps as a rough check. However, once the compression index and void ratio values have been determined to the designer's satisfaction, it will be found convenient to incorporate them in the bearing capacity index term suggested by the writer and to utilize the suggested form of equations. It should be noted that the equations in themselves have been rigorously developed. The results obtained, subject always to the basic assumptions, will be just as accurate as the values of the soil constants which are introduced.

Finally, it may be remarked that the writer does not consider it necessary to be able to determine the value of the bearing capacity index or of any other soil characteristic with hair-breadth accuracy in order to provide a satisfactory and logical basis for eliminating differential settlement in footing design. Using the equations and diagrams as intended, one would proportion all footings for a given settlement, say $\frac{1}{2}$ in. or $\frac{3}{4}$ in. If the value of the bearing capacity index is somewhat approximate, it may subsequently develop that the settlement may be somewhat more or less than predicted. The important consideration is that in such a case, differential settlement would still be negligible.

The discussion by Messrs. Goodman and Lee gives the writer the impression that on most points there are no major differences of opinion. A few brief comments may, however, be worth making:

As perhaps no more than a technicality, the writer takes exception to the discussors' statement than internal friction is dependent on soil density. Fundamentally, the friction coefficient is dependent only on the composition of the materials in contact and the condition of their surfaces. These characteristics are unchanged by variations in soil density. A research on this subject,[10] conducted some years ago by the writer, demonstrates, in fact, that while the values of peak and ultimate shearing resistance of a cohesionless soil are affected by soil density, the coefficient of friction of the material is independent.

10 "An Analysis of the Effect of Particle Interlocking on the Strength of Cohesionless Soil," by B. K. Hough, ASTM Bull. No. 176, September, 1951.

The discussors give the opinion that design for equal settlement is a worthy ambition but express doubts as to whether this objective can be reached in view of the lack of homogeneity of soil formations. To debate this contention is also to indulge in technicalities. However, the writer would personally choose to hammer out a number of basic principles which are inherently logical and consistent, and from such a base to deal as best he can with the inevitable deviations from the ideal which must be expected in all things human or natural, rather than to abandon the scientific approach and to deal always in expedients simply because of expected practical difficulties.

The discussors make several references to use of the theory of consolidation in connection with settlement analysis. They are consequently referred to the remarks on this subject which have been addressed to Messrs. Praszker and Scheer. It may also be observed that soil compression as distinct from consolidation as the latter term is commonly used, can and does occur in both fully and partially saturated soils. Hence, expulsion of water from the voids is not involved in all instances of soil compression as the discussors imply. In fact, when an architect has a choice, he will almost always select a site where ground water is at such depth that the bearing materials are not saturated.

Messrs. Goodman and Lee, like Mr. Harlan, have mentioned the difference between normally loaded and precompressed soil. They seem to feel that use of the consolidation theory, as they term it, for estimating settlements in precompressed soil is of questionable reliability. Once more the writer would like to emphasize that the essential requirement in analyzing settlements under any conditions is to know the slope of the field compression diagram over the range of anticipated loading. Distinctions as to the significance and the conventions for referring to the various branches of compression diagrams for laboratory test specimens have apparently clouded the thinking of a good many people on this subject. Whether a soil is normally loaded or precompressed it is most unusual when the field compression diagram as normally plotted is not linear from the point representing the initial loading and void ratio to the point representing the final condition after loading. The amount of compression which will occur is simply a function of the slope of this relatively short section of the diagram. If formalities can be waived to the extent of permitting the term compression index to be applied to the slope of this line regardless of the loading history, and if the compression index as thus defined is intelligently evaluated, settlement predictions for all types of soil should be equally dependable. Furthermore, this approach to the problem permits development of general equations such as those proposed by the writer for bearing capacity evaluation which apply as well to precompressed as to normally loaded soil formations.

With reference to the diagram Fig. GL1 presented by the discussors, the writer would like to point out that the full line curve marked "Hough (C = 70)" is not exactly as the writer would have drawn it. It is a basic premise in the writer's thesis that it is only under moderate loading that settlement is due chiefly to one-dimensional compression. For the sandy soil used as the basis for the illustration, the writer would consider a presumptive bearing value of about 3 tons per sq ft as given by most codes as the upper limit of "moderate loading." It is interesting to note that if a horizontal line at the value, p'_c = 6 kips per sq ft is substituted for the upper section of the curve, the full line (Hough) diagram and the dashed line (Terzaghi and Peck) diagram are in reasonably close agreement throughout. As pointed out in the remarks addressed to Mr. Peck, this raises a question as to the validity of the theory that lateral strain is a major factor in the settlement of foundations on sand since the writer

has demonstrated that all or nearly all the expected settlement can be accounted for by direct compression alone. Thus, there is considerably more difference in the theories and recommended procedures of the two methods that might appear from the discussors' comments.

The opening comments of Messrs. Kellogg and Murphree in respect to modern trends in structural engineering and the effect of these trends on foundation requirements are very similar to those made independently by Mr. Zetlin. The net effect of these comments is to give emphasis to the need for better understanding of the mechanics of settlement and for improving methods for reducing differential movements in structural foundations.

For an answer to the question as to the basis for the curves in Fig. 3, the discussors are referred to comments addressed to Messrs. Brown and Crawford who have raised the same question. It may be noted in passing that Eq. KM1 of the discussion by Messrs. Kellogg and Murphree proves to be a special case of a more general equation developed by the writer.[11] The equation quoted by the discussors is a slight modification of an equation developed, it is believed, by A. W. Skemptom which was intended to apply only to remolded clays. The writer's equation, namely, $C_c = a\ (e_o - b)$, is obviously in the same form, but it is in terms of e_o, the no-load void ratio of the soil rather than LL and is applicable to any soil type from coarse-grained sands and gravels to clays.

The writer is at a loss to understand the discussors' comment that use of Eq. KM1 yields C values much less than those shown in Fig. 3. A spot check has, therefore, been made to test the accuracy of Fig. 3, as indicated below. Assume a 40-blow clay with LL = 35%. Eq. KM1 gives a C_c value of 0.225 for this assumption. At 40 blows consistency, the water content might be between 15% and 20%. For a water content of 17.5%, the void ratio would be 0.47 if the clay is saturated. The bearing capacity index value would then be 65. This is in close agreement with the value shown in Fig. 3. The illustrative values all correspond closely with those taken from the writer's files for lacustrine clays from the New York State area. It is entirely possible that other clays have sufficiently different characteristics to require the plotting of a different curve in Fig. 3.

With reference to the balance of the discussion, the writer is impressed, for one thing, with the value of the digital computer in dealing with the equations of the paper and remembers with regret the many hours which he devoted to longhand calculations by the trial-and-error method. As for the relative merits of the discussors' Assumptions I and II, the writer cannot make any very conclusive contribution. However, one observation from general experience may be mentioned: It is very common to find that during construction, a foundation excavation, especially if it is large, will collect a good deal of surface water. If the excavation is in a clay soil, the clay will inevitably swell. When foundation elements designed for bearing on the clay in its original condition are constructed and loads applied, there is likely to be compression and settlement as the excess water is squeezed out. This could easily be attributed to rebound and recompression, but the writer is inclined to believe that far more often it is due to swelling and subsequent expulsion of water which has been soaked up. The bottom of a large excavation is normally a pretty wet place.

For the reason given previously, the writer is inclined to favor Assumption 1 so far as effects due solely to loading are concerned. It is conceivable that

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[11] "Basic Soils Engineering," by B. K. Hough, Ronald Press Co., New York, N. Y., 1957, p. 114.

one should also allow for swelling and the writer does not hesitate to confess that this idea has not previously occurred to him. Possibly allowance for swelling could be made by adopting Assumption 11 but the writer is reluctant to endorse this as standard procedure. In any case, a new and interesting possibility has been suggested by Messrs. Kellogg and Murphree.

Mr. Zetlin's support of the thesis that rupture theory is inappropriate as a basis for eliminating differential settlement of the foundations of a structure is appreciated. His authoritative comments on modern developments in structural design and related foundation requirements are both interesting and informative.

Mr. Zetlin has requested clarification of the basis for representing that there is a correlation between the bearing capacity index and the depth of significant stress as illustrated in Fig. 6 of the paper. This can readily be furnished: In non-mathematical terms it can be shown that in a general sense, the depth of significant stress varies with the total load. This follows from the well-known Boussinesq equation. However, if loading is to be restricted so that no more than a specified settlement will occur, then for a given soil which may be characterized by a given compression index, there is only one permissible loading and for this loading there is one particular value for the depth of significant stress. With a different soil, a different loading would be required to produce the same settlement. Thus when values of C and S are established, the depth of significant stress for a particular footing is also established.

Mathematically, depth of significant stress in a general sense is a function only of footing width B, contact pressure P_c and unit weight of soil γ as shown for example in Eq. 11 since the term h_s depends only on the relative magnitude of the initial body stress and the stress increment. In Eq. 14, however, the term H is the particular value of h_s for the limiting contact pressure p'_c. Again referring to Eq. 14 it can be seen that if the values of γ, B. S. and C are fixed, then the value of H is directly related to the value of p'_c. It was most interesting to the writer to discover that the p'_c versus H relationship for a wide variety of conditions is linear as illustrated in Fig. 6.

It is noted that the writer has unfortunately used the notation H both for the total physical thickness of a compressible soil stratum and also for a particular value of the depth of significant stress in unstratified soil as defined above. It is believed that this may largely account for Mr. Zetlin's difficulty in interpreting Fig. 6. It is believed that with this explanation there will be no difficulty in accepting the representation that for a given soil (with given C value) depth of significant stress increases with the settlement allowance since greater pressures are required to produce greater settlements. It is also logical that if settlement is held to a fixed value, depth of significant stress increases with the bearing capacity index since the greater the value of C, the greater must be the pressure to produce a given settlement.

In further reference to the presentation in Fig. 6, Mr. Zetlin is referred to the comments addressed to Mr. McClelland. It appears that the latter has spotted a section of the diagram which is somewhat unrealistic although consistent with the basic assumptions.

The writer is most favorably impressed with the methods described and evidently used by Mr. McClelland for indicating to a structural designer the footing sizes which should be used to insure approximately equal settlement. Referring to comments made by Mr. McClelland in his closing paragraph, the writer did not intend that the procedures developed in the paper, at least in their present form, should immediately be put in the hands of structural designers for direct use. For some time to come, it will be advisable for those with special training

and experience in the use of such procedures to make the analyses and calculations required to construct diagrams similar to Mr. McClelland's Fig. M1. However, this situation cannot last forever. Under the unrelenting pressure and economic necessity of cutting costs in engineering which on the whole is healthy rather than deplorable, procedures for footing design will be developed which are sufficiently reliable (foot-proof, if one wishes) and simple so that they can be used directly by structural detailers. This sort of gradual evolution has already taken place in structural engineering and in this field the hated "charts and tables" referred to with scorn by many writers are here to stay. There is no reason to suppose that the same development will not occur in foundation engineering which after all must be recognized as an extension of structural engineering. It will be the task of the present generation of specialists to assist in rather than to resist the development of the tools required for this purpose.

The writer is in agreement with Mr. McClelland's policy of distinguishing between sustained loads and transient or temporary loads especially on clay soils. This subject was not specifically mentioned in the paper and Mr. McClelland's comments are, therefore, appreciated.

Mr. McClelland has spotted a weakness in the presentation of material in Fig. 6 of the paper. It is quite unrealistic to suppose that sufficient loading would ever be applied to an 8 ft foundation element to create significant stress at a depth of 80 ft. The writer sincerely appreciates Mr. McClelland's courtesy in characterizing this representation as "unintended." The indication of significant stress at such considerable depth comes from calculating the loading which would be required to cause a compression of almost 2 in. in a virtually incompressible soil. In all probability, before such loading could be reached, rupture would occur unless the footing were surcharged to an abnormal degree. This is a good example of the need for a period of restricted use of new procedures before recommending their adoption in general practice.

Mr. Chinn has shown a very clear understanding of the basic principle of the new procedure proposed by the writer and has been less inhibited by old beliefs than some of the other discussors. His general endorsement of the paper and his encouragement and support are very welcome.

Mr. Chinn briefly mentions the generally accepted and often quoted assumption that the action of an element of granular soil beneath a foundation should be likened to the behavior of a triaxial test specimen and the theory that since such specimens bulge under loading, it is concluded that lateral strain must be the cause of settlement of foundations on sand. Mr. Chinn surmises, and rightly so, that the two situations are not in fact the same. He mentions that lateral pressure would increase with depth below the edges of a footing (thus progressively reducing the lateral strain). In the same vein, the writer would add the comment that the vertical compressive stress decreases significantly with depth beneath a foundation rather than remaining constant as in a test specimen. This is an added reason for assuming a decrease in lateral strain in the middle and lower sections of the zone of stress. Mention may also be made of the restraint imposed on lateral strain in the upper section of the zone of stress by friction between the soil and the foundation element. Thus, it is not difficult to find reasons for questioning the concept that lateral strains are the major cause of foundation settlement.

It is believed that most of the other comments made by Mr. Chinn have received consideration in remarks which the writer has addressed to some of the other discussors.

It appears that Mr. Focht and the writer are in disagreement on a number though not all the subjects covered in the paper. Mr. Focht like Mr. Harlan, seems to think that most foundations are designed by engineers who have had special training in this work and who are, therefore, presumably well informed on the need for due consideration of both rupture theory and settlement. As noted in remarks addressed to Mr. Harlan, although this might be desirable, it is simply not the situation which exists today in general practice. Therefore, it makes little difference whether the engineers who design a relatively small percentage of structural foundations are "careful" or "careless" readers of the available texts as long as most of the foundations are designed by people who have never even seen the referenced texts. Furthermore, it is somewhat less than satisfactory to engage in a "careful" reading of a text which, although it may discuss settlement as well as rupture theory, presents the wrong explanation of the cause of settlement.

Mr. Focht is evidently most exercised about the inclusion of Fig. 3 in "what is otherwise a generally logical paper." He says the inclusion of this figure invites misinterpretation and misapplication of the paper. This may be true to some degree as it is probably also true of anything ever written or said or published about the use of standard penetration resistance values. Correlations between penetration resistance values and various soil coefficients have now appeared in quite a number of reputable publication, however, and it would seem to the writer that the time has come for Mr. Focht to reconcile himself to the fact that since these values are useful under certain reasonably well known conditions, it must be expected that they will be referred to with increasing regularity in future publications. The International Society for Soil Mechanics and Foundation Engineering has in fact created a Sub-Committee on Static and Dynamic Penetration Test Methods. This well-sponsored committee is currently investigating use of the standard penetration resistance test throughout the world and will probably submit a report at the Fifth International Conference in 1961.

One of Mr. Focht's many objections to the use of N values is that it may lead to "handbook" designs. By implication at least it would appear that Mr. Focht considers "handbook" designs inherently objectionable. It would be interesting to speculate on how many less structures there would be in the world today or how many of the existing structures would be less strong or serviceable if it were not for the availability and use of large numbers of very excellent handbooks, design charts, and tables. The implication that a "handbook" design is necessarily bad is one which in the writer's opinion is not supportable in the light of practical realities. It is hoped that Mr. Focht did not intend this implication.

For purposes of this discussion, the writer submits that regardless of the merits of his opinions of the value of handbooks and penetration resistance records per se, Mr. Focht is somewhat extreme in his references to Fig. 3 of the paper. It is to be noted that the writer specifically states that Fig. 3 is intended only to show "the order of magnitude" of the bearing capacity index and the extent to which index values are affected by the in-place density or consistency of the soil. The discussion by Messrs. Brown and Crawford suggests that the illustrated correlation goes considerably beyond this modest objective. Furthermore, as the writer has pointed out to Mr. Harlan, the reader is under no obligation to use penetration resistance values at all if he prefers and is in a position to evaluate the bearing capacity index by reference to test data. For further discussion one is referred to the comments to Mr. Harlan.

In the balance of his discussion, Mr. Focht draws up a formidable list of topics which he states or implies have not been given due consideration, in consequence of which the paper is alleged to have a number of shortcomings or "errors." In this connection, it would be interesting to know Mr. Focht's views, for example, on the validity of the Boussinesq equation as applied to problems in foundation engineering. As is generally known, this equation was developed for elastic solids, not particulate materials, and most especially not for loose combinations of matter in solid, liquid, and gaseous states. Would not Mr. Focht's viewing-with-alarm of the possible effects of non-uniformities in the soil apply with even greater force to the theories of M. Boussinesq than to the modest efforts of the writer? Yet we find that use of the Boussinesq equation even in the field of foundation engineering has become standard practice and can be justified, at least pragmatically. It turns out, as is so often the case, that many of the imagined difficulties never develop and those which do are far less serious than anticipated.

Mr. Focht is requested to consider that the writer is advancing a new theory or concept, namely that soil compression, not lateral strain or displacement, is the chief factor in the settlement of foundations under moderate loading. In advancing a new theory it is certainly not unusual to limit the development and indicated application to relatively simple conditions. It verges on the unreasonable to condemn the entire paper because it does not provide a design procedure for every possibie combination of conditions.

It would be unfair to imply that Mr. Focht condemns the entire paper. On the contrary, he specifically comments on the value of the theoretical development of Eqs. 7 through 37 and Figs. 5 through 8. It is the writer's hope that from this and similar developments of basic theory, rational design procedures can eventually be devised which will be found acceptable in general practice.

INVESTIGATION OF UNDERSEEPAGE, MISSISSIPPI RIVER LEVEES[a]

Closure by W. J. Turnbull and C. I. Mansur

W. J. TURNBULL,[1] F. ASCE AND C. I. MANSUR,[2] F. ASCE.—The purpose of this paper was to demonstrate the field and laboratory procedures for investigating the underseepage problem beneath Mississippi River levees.

In his discussion, Mr. Suter has questioned the validity of the formulas and computations presented in the paper because, in his opinion, the basic source of seepage is ground water flowing toward the river from the hill or valley wall rather than from the river. During periods of low river stages the ground-water flow in alluvial valleys is toward the river and down the valley. However, during sustained high flood stages the direction of seepage flow in a pervious aquifer beneath a levee is from the river landward. This is known from the fact that water or seepage flows from a point or line of high potential toward an area of lower potential. The direction of seepage flow beneath levees during periods of flood stages has been measured by the writers with piezometers many times; the slope of the hydraulic gradient beneath the levee during flood stages of the Mississippi River, and correspondingly the direction of seepage flow, has always been from the river toward the landside. It is to be noted that this condition exists in a relatively wide alluvial valley. In narrower valleys it is easily possible that the land surface may slope rather steeply toward the river, and, in consequence, the ground-water gradient toward the river may be fairly steep. In this case, as the flood stage rises against the levee there would be two opposing gradients, one the ground water toward the river and the other seepage from the river, with a low potential point between. Flow from either side would be toward the low point of potential at any given time regardless of where it was located. This would exist until the low potential point was ironed out by one gradient overcoming the other. The design of a pressure relief system for this last-described case would, of course, take into consideration all the physical conditions existing at the site.

The condition described immediately above certainly does not apply generally on levees along streams in relatively flat and wide alluvial valleys; instead, the condition of flow from the river landward does apply at flood stages where underseepage and consequent excessive pressures become a problem.

Chemical or physical tests on seepage water collected at the toe of a levee during flood stages are not necessarily indicative of the source of pressure causing seepage flow beneath a levee. The reason seepage water collected at the toe of a levee can have the characteristics of the normal ground water rather than of the river water is that most floods are of such relatively short duration that there is not enough time for water from the river to flush out the ground

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[a] September, 1959, by W. J. Turnbull and C. I. Mansur.

[1] Chf., Soils Div., Corps of Engrs., Waterways Experiment Station, Vicksburg, Miss.

[2] Engr., Hvy. Constr. Div., Fruin-Colnon Contracting Co., St. Louis, Mo.

water in existence beneath the levee before the river stage falls. For example, the average velocity of seepage flow beneath a levee for the following conditions

k (coefficient of permeability) = 1000×10^{-4} cm per sec = 0.20 ft per min
i (hydraulic gradient) = 0.025
n (porosity) = 0.40

would be

$$v \text{ (seepage velocity)} = \frac{k\,i}{n} = 0.20 \times \frac{0.025}{0.40} = 0.0075 \text{ ft per min} = 10.8 \text{ ft per day.}$$

Thus, during a flood with a high stage for 20 days, seepage from the river would have traveled only about 200 ft in a very permeable stratum.

Consequently, it is firmly believed that the foregoing verifies that the fundamental assumption of landward flow under the levees during flood stages is sound for the greatest majority of Mississippi River levees and can easily be proved.

CONSTRUCTION MATERIALS CONTROL - AASHO ROAD TEST[a]

Closure by James F. Shook

JAMES F. SHOOK.[9]—The contribution by Mr. Abdun-Nur is most welcome. He has pointed out an important fact--that to most effectively apply statistical controls one must recognize the problems of the contractor. Not only must such control procedures be designed to assure the desired quality without delaying the contractor, but they must be applied intelligently and in a clearly stated manner. Preferably, specifications should state how such techniques will be applied and how they might affect the operations of the contractor.

Mr. Abdun-Nur inquired about the use of nuclear density measuring equipment for layers thicker than 3 in. to 4 in. The surface-type gage used on the AASHO Road Test was not adaptable to use with greater depths of material; neither are presently available commercial gages. Depth gages, designed to be inserted through tubes to most any depth below the surface, are available; however, the writer has had no experience with them.

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[a] October, 1959, by James F. Shook.
[9] Materials Engr., ASSHO Road Test, Highway Research Board, Ottawa, Ill.

SUBSURFACE EXPLORATIONS IN PERMAFROST AREAS[a]

Closure by James R. Cass, Jr.

JAMES R. CASS, JR.,[1] M. ASCE.—It is believed that the sharing of experiences through the medium of discussion, has enhanced the value of the original paper.

The use of test pits as described by Mr. Johnson is an excellent method for shallow investigations up to about 20 ft in depth; however, for deeper explorations, some method of drilling still needs to be developed that will use a minimum of heavy or special equipment. The use of refrigerated compressed air mentioned by Messrs. Lange, Stevens, and Verville probably requires substantial special equipment in addition to the basic drill rig. Because transportation is a major problem in the arctic, it is readily apparent that great advantage will be derived from the development of a method of subsurface exploration in permafrost utilizing a limited amount of equipment which can be readily shipped to remote areas.

It is suggested by the discussion that the season of the year has some effect on the success of the exploration program. The work described in the paper was undertaken during the late summer months when the active layer was fully thawed. More satisfactory results probably would be obtained if the work had been done in the early spring with the ground frozen to the surface.

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[a] October, 1959, by James R. Cass, Jr.

[1] Engr., Fay, Spofford & Thorndike, Inc., Boston, Mass.

CONSTRUCTION PORE PRESSURES IN AN EARTH DAM[a]

Closure by C. Y. Li

C. Y. LI,[1] M. ASCE.—The assumption by Mr. Moore of an increase in the effective stresses is correct only if there were a significant amount of downward seepage pressures in the embankment. According to the patterns of the construction pore pressures, the pore water drained slowly outward to the shells as well as slowly downward to the rather pervious foundations. The writer doubts that the downward seepage pressure could be appreciable. The water level in the embankment at the settlement installation, as shown in Fig. 7, was that in the 2-in. pipe of the settlement apparatus. It may not be the measured water table during construction. The pore water was draining into the pipe at that time. This water level was receding after completion of the dam, and became stabilized at about El. 2087 m close to the phreatic line during operation of the reservoir.

To the writer's knowledge, there is yet no satisfactory approximate method to calculate pore pressures for practical use based on laboratory consolidation tests. The factors influencing the set-up of pore pressures are numerous and complex. The observation at Quebradona Dam indicates that one of the most important factors concerning pore pressures is drainage or dissipation which cannot be ignored during calculation.

Mr. Moore reported that, at Adaminaby Dam, the laboratory compression curve is quite different from the field compression curve. This should be expected because, among other factors, the drainage in the field is quite different from the one-dimensional drainage during the consolidation tests.

As stated, the writer's proposed method of calculating pore pressures is to include the effect of drainage by an approximation of the gradual, continuous drainage process by sudden, arbitrary chosen steps. It is to some degree similar to arithmetic summation of an integration problem. Mr. Yachin used 2.5 psi effective stress increment and 1/3 dissipation to calculate and compare with the results of calculation by the author using 5 psi effective stress increment and 1/3 dissipation. It may be expected that the results would not be the same. But the dissipation factor is an average value for the chosen load increment and may not be considered a constant when the load changes.

Mr. Yachin's calculation of pore pressures by using Terzaghi's one-dimensional consolidation theory is very interesting. But it is difficult to conceive that the basic assumptions of the consolidation theory can be approximately realized in the field drainage of pore water and that the effect of the trapped air in the soil voids on pore pressure is negligible.

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[a] October, 1959, by C. Y. Li.

[1] Civ. Engr., Gannett Fleming Corddry and Carpenter, Inc., Harrisburg, Pa.

COMPACTION OF SANDS AND BEARING CAPACITY OF PILES[a]

Closure by G. G. Meyerhof

G. G. MEYERHOF,[1] F. ASCE.—The interesting results of Mr. Schiff's experiments with groups of model piles are in general agreement with those of earlier investigators. While some of the differences observed can be explained by the heterogeneity of the soil used, the small group settlement in relation to that of a single pile is due to the absence of an effective pile cap (or a pile cap clear of the ground) in these tests, as shown by the author's analysis.

Thus, for a given number of piles, in groups without cap (or free standing groups) the ultimate bearing capacity per pile and the group settlement under a given pile load increase as the pile spacing is reduced below about 8 times the base diameter, because the individual soil compaction and pressure zones overlap. Furthermore, for a pile spacing less than approximately 3 times the base diameter, the individual soil shear failure zones overlap, leading to soil arching between the piles; the pile group acts then as an equivalent block footing so that the ultimate bearing capacity per pile and the settlement under a given pile load decreases with a smaller pile spacing. These results obtained from the writer's analysis are supported by the pile tests mentioned in the paper and by subsequent investigations.[2]

In practice, pile groups usually have caps resting on the ground (piled footings or rafts), so that a group acts as an equivalent block foundation irrespective of the pile spacing. In that case, for a given number of piles, the ultimate bearing capacity per pile in the group and the settlement of the group under a given pile load, increase with a greater pile spacing at a decreasing rate. Moreover, both this group bearing capacity and settlement are greater than for a single pile, as was indicated in the paper.

Mr. Szechy and Mr. Nishida have estimated the increase of skin friction of single piles by soil compaction, in the first method by assuming the diameter of the compaction zone, and in the second method by estimating this diameter on the assumption of an analogous expanding cylinder. While their results are of the same order as those obtained by the writer, he has shown in his paper that the diameter of the compaction zone along the shaft of driven piles can be estimated from and is actually governed by the stresses induced in the soil near the base, because during driving of piles the tip (or casing of a displacement caisson) penetrates the soil throughout the embedded length of the final shaft. It should also be remembered that the bearing capacity of single piles and especially displacement caissons in cohesionless soils is largely due to

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[a] December, 1959, by G. G. Meyerhof.

[1] Head, Dept. of Civ. Eng., Nova Scotia Technical College, Halifax, N. S., Canada.

[2] "Notes on the Behaviour of Model Pile Groups in Sand," by J. G. Stuart, T. H. Hanna, and A. H. Naylor, Proceedings, Symposium on Pile Foundations, Sixth Internatl. Congr. Bridge and Struct. Enrg., Stockholm, Sweden, 1960, p. 97.

base resistance and the skin friction is relatively small. The base resistance becomes even more important in pile groups, when the skin friction could well be ignored.[3]

Messrs. Kantey and Low have added welcome field data on the compaction of cohensionless soils near groups of driven piles and Franki caissons. Their results confirm the writer's findings that the ratio of final to original cone penetration resistance near the piles decreases as the original density (or cone resistance) of the sand increases so that pile driving in soils of variable density achieves a more uniform compaction. Local variations of the observed ratio of final to original cone resistance and in the size of the compaction zones can probably be explained by some heterogeneity of the soil and variations of the energy per blow of the hammer during the pile driving or installation of the caissons. In this way the observations support the writer's conclusions that the state of compaction produced by pile driving in cohesionless soils depends partly on the physical properties of the soil, mainly the initial relative density and ground-water conditions, and partly on the characteristics of the foundation, mainly type and layout of piles or caissons and the energy of the hammer used in their installation.

The writer is grateful to the various discussors for their helpful and encouraging comments, which have added valuable information on a subject of great practical interest and considerable complexity.

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[3] "The Design of Franki Piles with Special Reference to Groups in Sands," by G. G. Meyerhof, Proceedings, Symposium on Pile Foundations, Sixth, Internatl. Congr. Bridge and Struct. Enrg., Stockholm, Sweden, 1960, p. 105.

DYNAMIC TESTING OF PAVEMENTS[a]

Discussion by O. M. Christof Ehrler

O. M. CHRISTOF EHRLER.[34]—The authors' work is a contribution to the solution of the problem of soil dynamics that is highly welcomed. There is a need for special attention because in spite of numerous recent attempts to use the theory of elastic continuum, authors continue to apply the simple spring-mass system which is easy to study. Though the simple spring-mass theory does not duplicate actual conditions a clear concept of the behavior is obtained. Also, the theory will probably show enough agreement with the actual system for practical purposes.

A review of recent literature on soil dynamics shows the tendency of some authors to ascribe discontinuities in the relationships between frequency and certain "constants" of the system and to renounce a closed homogeneous function. It is remarkable to note that the authors make such an assumption. In analyzing experimental investigations made at the Institute for Foundation Engineering and Soil Mechanics of the Technische Hochschule of Dresden, Germany, the writer observed a relation between frequency and mass of the system similar to the variation of mass found by Messers. Heukelom and Foster, with an asymptodical approximation to the mass of the vibration machine for higher frequencies. This phenomenon is important in the analysis of the dynamic behavior of basements and the writer plans to publish information on this subject in the future. It would be of interest to know the numerical values for the constant and the variable masses used by the authors for their computations, especially the proportion of the constant mass at low frequencies to the mass of the vibration machine.

Further it is considered worthy of note that the investigations included measurement of amplitude and phase angle. For many reasons phase angle often is neglected in analyzing the system considered, though it facilitates analysis and enhances the value of the data.

Certain questions with respect to the mathematical foundation of the derivations used by the authors would be of interest in further research. First there is the problem of elastic stiffness R and its dependence on the dynamic deflection of the medium. This dependence is shown experimentally by the research work reported in the paper, although the magnitude of the variation is not constant. Since the dependence is influenced to some extent by the form of solution of the differential equation of vibrating system, a statement would be of interest as to whether the authors observed such an effect and if so, if it was too small to report.

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[a] February, 1960, by W. Heukelom and C. R. Foster.

[34] Dipl.-Geophysiker, scientific collaborator, Institut fur Grundbau und Baugrundmechanik, Technische Hochschule, Dresden, Germany.

There is the question, too, of the influence of a variable mass found experimentally on the solution of the underlying differential equation.

The third question concerns the problem of damping. The authors have demonstrated in the charts, that phase angle ϕ is very small in the frequency range investigated and therefore damping can be neglected. Are there investigations on the range of validity for this assumption, too? The authors mention temporary strains under traffic conditions of 0.1 sec to 0.01 sec, that would correspond to frequency conditions of 10 cycles to 100 cycles per second. Fig. 10 includes frequencies up to 50 c/s only. Because there may be frequencies with phase angles of 90° and 180°, and if only phase angles below 90° were considered, it seems to be evident that the natural frequency was not reached. Possible reasons may be the decrease of mass of system with raising frequency and the relatively small mass of the vibration machine compared with the high elastic stiffness of the pavement and the subsoil tested which result in a high natural frequency.

The experimental investigations noted previously in practically every case passed a real natural frequency within the range tested in spite of a decreasing mass. In these tests a relatively large mass (about 3 tons of weight) was placed directly on the bare ground which was only moderately stiff. The observed greater damping effect raises the question of the type of damping that is occurring.

The writer considers the authors' article to be of great interest for future work and one that will provide some knowledge of the problems mentioned. The authors should be congratulated for their promising attempt in this highly complex field, especially for their very interesting observations on wave velocities.

SOIL STRUCTURE AND THE STEP-STRAIN PHENOMENON[a]

Discussion by B. K. Hough, A. A. Eremin

B. K. HOUGH,[19] F. ASCE.—The paper by Messrs. Trollope and Chan is evidence that the need for review and revision of some of the concepts of soil structure developed to date in the field of engineering is at last gaining recognition. This awakening is due to the gradual realization that in certain respects, engineering concepts of structure are not in agreement with the vast body of theory and information developed and generally accepted in the soil sciences and other similar areas. Since the discussion and references given by the authors are, for the most part, limited to work undertaken recently in the field of engineering, the following account of some of the background material is offered to give a broader perspective on this new and very important development.

As a preliminary, it may be said that understanding of structure requires some knowledge of at least two features of soil-water systems. One is the physical character and composition of the soil particles themselves, particularly particles of secondary minerals in the colloidal size range. The other is the existence and nature of the forces which operate between the particles in clay-water systems.

In the field of soil mechanics it was at first supposed that clay and silt particles resembled grains of sand and gravel in respect to shape, specifically in having a bulky shape, such as, rounded or angular.

The well known hypothesis of clay structure referenced by the authors is an example of this early and erroneous belief. In soil mechanics, this belief and a great many theories associated with it has persisted until recently, as evidenced by the fact that it is presented without significant change or qualification in engineering textbooks published as late as 1953. In other fields, however, the concept that clay particles are typically of platey shape has been generally accepted for some time.

This concept derives logically from information on the crystal structure of clay minerals and their tendency toward basal cleavage, information originating in the general field of geology. R. E. Grim,[20] one of the foremost contributors to present knowledge of clay mineralogy, traces the origin of the clay-mineral concept, that is, the belief that clay particles are crystalline in nature, at least as far back as the work of LeChateller in 1887. Grim also cites the work of Hadden in Sweden, Rinne in Germany, Marshall in England, and Bradfield, Henricks, Ross and many others in this country during the 1920's in the field of clay mineralogy, and states that the clay-mineral concept was generally accepted in about 1930. Since 1930, the development of such techniques as x-ray

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[a] April, 1960, by D. H. Trollope and C. K. Chan.

[19] Cons. Engr., Ithaca, N. Y.

[20] "Clay Mineralogy," by Ralph E. Grim, McGraw Hill Book Co., Inc., New York, N. Y., 1953.

diffraction and electron microscopy and their use by workers in many fields in a host of university, commercial, and other laboratories are noted as making possible the advancement of knowledge of crystal structure and individual clay particle shape.

The second essential consideration in developing acceptable concepts of clay structure, namely a knowledge and understanding of the forces that operate between particles in a clay-water system, has a similar geneology. Some of the present theory dates back to the work of Gouy in 1910, Chapman, Stern, Wiegner, Langmuir, Freundlich, Mattson, and more recently Schofield, Verwey, Overbeek, van Olphen and others. The work of these physicists and chemists resulted in the development and general acceptance of the diffuse double-layer theory which is applicable to systems made up of an electrolyte and particles having a net surface charge. The existance of such a charge on or near the surface of clay particles was established long before the crystalline nature of clay was known. Thus the double-layer theory, which is applicable to many systems, was found to have application in clay-water systems and to provide a basis for explaining many aspects of observed electrokinetic behavior in clays, and more recently, swelling.

Much of the early research involving applications of the double-layer theory to soils was done by agronomists and was conducted to provide information on dispersion, flocculation and aggregation of particles, rather than soil structure as the term is used in engineering. Bradfield, Baver, Jenny, Schofield, M. B. Russell, Holmes, and many others made some of the original contributions to this subject.

Discovery and some realization of the significance of this wealth of material in engineering applications came about during fundamental research at Cornell University in the period 1946 to 1951 for an Army-sponsored project in the field of soil solidification. The research team engaged on this project was purposely recruited from the fields of agronomy, chemistry, physics, ceramics, geology, and chemical engineering in addition to civil engineering, in the hope that this combination of skills and experience would be more productive than the efforts of a number of workers from engineering or any other single field. The final report on this project is one of the first publications in the soils engineering literature to describe the material available from other fields and to show its application in engineering problems. Publications[21,22,23] by individual members of the Cornell research staff contain further details of the work and are more readily available than the final report to the Army.

Credit for some of the first quantitative tests of the basic scientific principles with particular reference to the compressibility of pure clays, together with refinements and critical evaluation of the theory, must be given to G. H. Bolt. In his doctoral thesis,[24] Bolt verified Schofield's formulation of ionic (double-layer) forces in relation to the compressibility of clay-water systems.

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21 "The Relation Between Exchangeable Ions and Water Adsorption on Kaolinite," by A. G. Keenan, R. W. Mooney, and L. A. Wood, Journal of Physical and Colloid Chemistry, Vol. 55, 1951, p. 1462.

22 "Adsorption of Water Vapor by Montmorillonite. I. Heat of Desorption and Application of BET Theory," by A. G. Keenan, R. W. Mooney, and L. A. Wood, Vol. 74, 1952, p. 1367.

23 "Adsorption of Water Vapor by Montmorillonite. II. Effect of Exchangeable Ions and Lattice Swelling as Measured by X-ray Diffraction," by A. G. Keenan, R. W. Mooney, and L. A. Wood, 1952, p. 1371.

24 "Physico-Chemical Properties of the Electric Double Layer on Planar Surfaces," by G. H. Bolt, Ph. D. Thesis, Cornell Univ., Ithaca, N. Y., 1954.

According to Schofield, the electric double-layer in clay-water systems operates to create an osmotic pressure between clay particles with obvious effects on their spacing and orientation. The treatment of this situation in terms of osmotic pressure of double-layer ions was an important forward step in comparison with earlier discussions in terms of zeta potential. In this development of the subject, the operation of an external counter-balancing force, said by the authors to have been ignored, was clarley inferred and was an independent variable in Bolt's experiments. The publication of Bolt's thesis was followed by a series of other publications[25,26,27,28,29,30] on related subjects, all having application to soil structure. Schofield, van Olphan and others demonstrated the nature of the electrostatic edge-to-face bond of the "card house" structure, mentioned by the authors. A very important extension of the basic material to development of theories relating to the freezing of pore water and resultant frost-heaving effects has been made by R. D. Miller.[31,32] In 1957, the writer included excerpts from the contributions of some of these men in his textbook[33] and showed the application of the osmotic pressure theory to soil structure and in consequence to soil compressibility and shearing strength.

It is the writer's conclusion that the most promising plan for improvement of the present thinking and knowledge in soils engineering, especially in matters relating to the vital subject of soil structure, is to continue collaboration with the pure sciences and to tap the wealth of material available to us in fields outside our own. It is believed that soil physicists and chemists working in the field of agronomy are probably best qualified for such joint studies with engineers, and it is to be hoped that mutually satisfactory arrangements for cooperative research with these men can and will be made.

Although a sincere believer in the importance of continuing research in matters affecting soil structure and in the need for collaboration with others outside the field of engineering, the writer has been advised not to expect that such studies will necessarily lead to complete revision of all present engineering concepts. There is already evidence that some of the new concepts described by the authors are applicable only in the most finely divided pure clays, such as those with Stokes Diameters of 0.2 microns and smaller. Thus, it may be that engineering concepts need revision only in certain, rather inferquent applications. However, the suggested research would still be of great value even

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25 "Compression Studies of Illite Suspensions," by G. H. Bolt and R. D. Miller, Proceedings, Soil Science of America, July, 1955.

26 "Physico-Chemical Analysis of the Compressibility of Pure Clays," by G. H. Bolt, Geotechnique, June, 1956.

27 "Analysis of the Validity of the Gouy-Chapman Theory of the Electric Double Layer," by G. H. Bolt, Soil Science.

28 "Conditions Affecting Formation of the Montmorillonite-Polyacrylic Acid Bond," by B. P. Warkentin, G. H. Bolt, and R. D. Miller, Soil Science, January, 1958.

29 "Calculation of Total and Components Potentials of Water in Soil," by G. H. Bolt and R. D. Miller, Transactions, Amer. Geophysical Union, October, 1958.

30 "Tactoid Size and Osmotic Swelling in Calcium Montmorillonite," by A. V. Blackmore and R. D. Miller, Proceedings, SSSA (in press).

31 "Particle Size, Overburden Pressure, Pore Water Pressure and Freezing Temperature of Ice Lenses in Soil," by R. D. Miller, J. H. Baker, and J. H. Kolaian, Transaction VIII, Internatl. Congress Soil Science (in Mss).

32 "The Role of the Electrical Double Layer in Frost Heaving," by L. A. Cass, M. S. Thesis, Cornell Univ., 1958.

33 "Basic Soils Engineering," by B. K. Hough, Ronald Press Co., 1957.

if this were the eventual conclusion, since it would materially strengthen present beliefs in regard to fundamental properties of soils.

A. A. EREMIN,[34] M. ASCE.—The steps in the stress-strain diagram are characteristic of colloidal particles and the shape of the steps may be used in the quantitative analysis of colloidal particles and soil grain particles. The mechanical properties of the colloidal soil, however, will vary not only with the quantitative relation of the colloidal particles and the grain particles but also with the chemical properties of the colloidal particles, the variation in temperature of the soil, and the ionic properties of colloidal particles. Obviously, computation of bearing properties of colloidal soil cannot be based on the stress-strain diagram without a complete study of the physical properties of the colloidal particles.

The authors should receive credit for their interesting discovery in the stress-strain relationship of colloidal soil.

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[34] Assoc. Bridge Engr., California State Highway Dept., Sacramento, Calif.

COMPUTER ANALYSIS OF SLOPE STABILITY[a]

Discussion by Bobby Ott Hardin

BOBBY OTT HARDIN,[6] A. M. ASCE.—The author has presented a valuable addition to the library of programs dealing with slope stability. To the writer's knowledge, a computer program for the analysis of the stability of slopes that will handle every soil situation that could arise in practice has not been written. Such a program would indeed be voluminous and probably not worth the effort required to produce it. The program presented by the author has some novel features that make it valuable as well as restrictions that make it non-applicable to certain situations which may arise.

The distinguishing feature of the author's program is that the surface and strata division lines can be broken lines defined by several coordinates. This makes the program quite applicable to the analysis of the stability of earth dams. However, in the analysis of slopes arising from cuts through or fills resting on natural earth deposits, there will often exist three or four approximately horizontal layers of soil which will vary in both strength characteristics and unit weight. Such a slope is shown in Fig. 13. The program presented by the author will not analyze such a problem.

The writer has written a program for the IBM 650 digital computer that will analyze problems having one or more soil layers. The following is a description of that program.

The program was written using the Bell II interpretative system and uses approximately 400 Bell II instructions plus an additional 300 memory locations (the required number of memory locations could easily be cut to approximately 100). The interpretative system requires approximately 1,000 locations. Hence the entire program can be accommodated by the 2,000 word storage capacity of the computer's drum.

The program, interpretative system, and data for the problems to be solved are all punched into cards and fed into the computer in this manner. The answers to the problems are punched from the machine into cards and can then be typed out. Since this computer is widely used the program here presented is quite practical.

The assumptions made are the same as those listed by the author except that the four layers may have different unit weights whereas both soils in the author's program must have the same unit weight. The features of the program are as follows:

1. The surface is composed of two horizontal and one sloping line segments with the angle designated by the user. The author's program is more general in this respect.

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[a] June, 1960, by John A. Horn.

[6] Asst. Prof. of Civ. Engrg., Dept. of Civ. Engrg., Univ. of Kentucky, Lexington, Ky. Presently studying on Natl. Science Foundation Fellowship at Univ. of Florida, Gainesville, Fla.

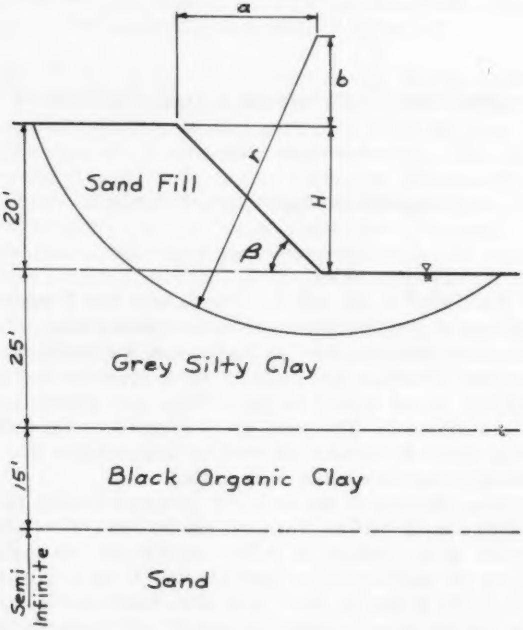

FIG. 13

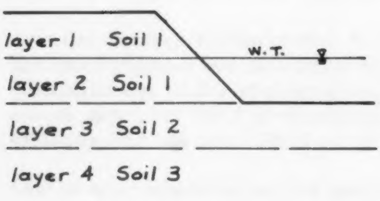

FIG. 14

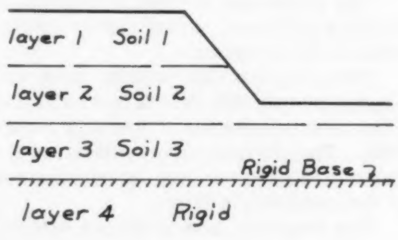

FIG. 15

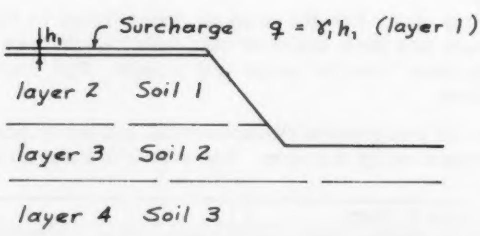

FIG. 16

2. The program can handle up to four layers of soil, each layer being horizontal and having different values of thickness, unit weight, cohesion, and friction. The thickness of the bottom layer is not designated and it is considered to extend beyond the deepest assumed failure surface. Here the writer's program is less restricted than the author's.

3. A horizontal water table can be specified by making it one of the strata division lines and using the ordinary unit weight above and submerged unit weight below. Hence, if the water table does not fall on a division between two soil types, three different soils and the water table make up the four layers (Fig. 14).

4. A horizontal rigid base can be specified by assigning to the layer below this base the large values of cohesion and friction which a rigid base would possess. Hence the rigid base will be one of the layers and there can be three additional layers or soil types (Fig. 15).

5. A uniform surcharge can be simulated by assigning to the top layer a large value of unit weight and small value of thickness with zero strength such that the unit weight and thickness combine to produce the desired amount of surcharge. This leaves three layers of soil (Fig. 16).

6. Values of cohesion and tan ϕ up to 1049 units with any consistant set of units can be used.

7. Tension cracks can be handled by treating the soil above the cracks as surcharge.

8. The program will solve any number of problems in succession.

The program was originally written to punch out the factor of safety for each slice assumed. However, a great saving in time was gained by using a searching technique similar to that used by the author. This technique was added after reading the author's paper.

Notation.—In using the program, the following data for each problem are punched into data cards, six values per card, and are fed into the computer behind the program.

a_o = the initial value of a;
b_o = the initial value of b;
N = number of slices into which each failure mass is divided;
δS = small increment between centers of failure circles;
ΔS = large increment between centers of failure circles;
r_o = initial value of r;
R = limiting value of r;
Δr = increment in r;
β= slope angle;
H = height of slope;
γ_1, γ_2, γ_3, γ_4 = unit weight of soil;
C_1, C_2, C_3, C_4, = cohension of soil;
h_1, h_2, h_3, = thickness of layer; and
ϕ_1, ϕ_2, ϕ_3, ϕ_4 = friction angle of soil.

Using the center specified by a_o, b_o, the factor of safety is computed for a series of radii between the limits r_o and R differing by increments, Δr, and the minimum factor of safety for this series of failure circles is stored. The machine then chooses another center and computes the minimum factor of safety for the next series of failure circles corresponding to the same variations in radius. The minimums for each of the two center are compared and the

least stored. The relative value of the two minimums signals the computer as to which center to assume next. The computer thus searches for the center with minimum factor of safety as described by the author. When the minimum factor of safety is found, it is punched out along with the coordinates of the center and radius of the failure circle and the value of the overturning and resisting moment. The original data for the problem is also punched along with the answer. If the data for another problem is in the card hopper of the computer at this time, it is read into the computer and the next problem solved.

The IBM 650 using the Bell interpretative system is much slower than the Illiac. The time required for the writer's program depends upon the problem and value chosen for N and the initial values a_o and b_o. However, the ordinary problem will seldom run more than 45 min.

It should be mentioned that in certain cases where water table, rigid base, and surcharge exist at the same time, the writer's program would be limited to one soil type. However, as it is written, the writer's program could easily be modified to handle more than four layers, perhaps five or six, and still be accommodated by the drum of this computer if the need should arise.

The writer is aware of the fact that other programs for the analysis of slope stability have been written that will perhaps deal better with certain problems than either of the programs here discussed. However, it is felt that much is to be gained by the publication of these programs such as has been done by the author.

The writer wishes to express his appreciation for the cooperation of Mr. H. A. Meyer and the staff of the University of Florida Computing Center.

ERRATA

Journal of the Soil Mechanics and Foundations Division

Proceedings of the American Society of Civil Engineers

June, 1960

p. 4. In Eq. 2 change s to S and C to c. In Eq. 3 change C to c.

p. 88. The material indentified as Fig. 4 should be deleted and should be replaced by the following:

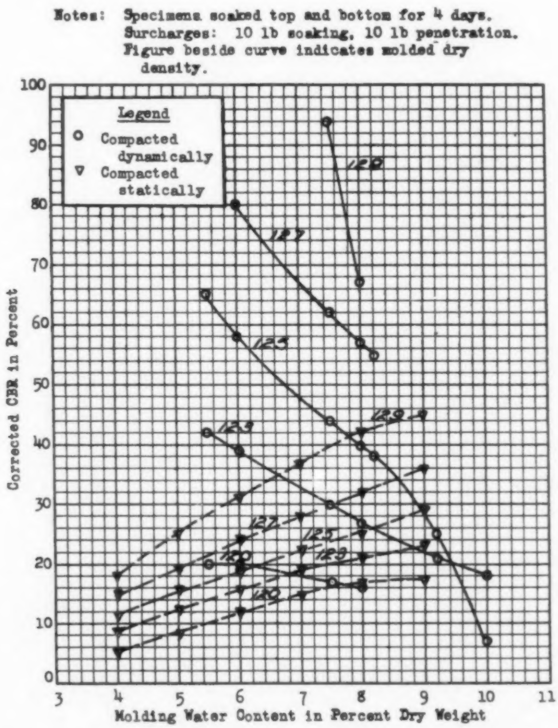

FIG. 4.—VARIATION OF CBR WITH DENSITY AND WATER CONTENT

August, 1960

p. 38, line 1 under the heading "Diagrammatic Representation" change "can" to "cap."

p. 44, line 14; p. 61, line 5; p. 61, line 4 below Eq. 29; p. 61, line 6 below Eq. 29. Change Eq. 24 to Eq. 27.

p. 47, line 3. Delete the word "one."

p. 61, line 7. Change "25, and 26" to "28, and 29."

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 2270(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural Division during 1959.

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OCTOBER: 2189(AT4), 2190(AT4), 2191(AT4), 2192(AT4), 2193(AT4), 2194(EM4), 2195(EM4), 2196(EM4), 2197(EM4), 2198(EM4), 2199(EM4), 2200(HY10), 2201(HY10), 2202(HY10), 2203(PL3), 2204(PL3), 2205 (PL3), 2206(PO5), 2207(PO5), 2208(PO5), 2209(PO5), 2210(SM5), 2211(SM5), 2212(SM5), 2213(SM5), 2214 (SM5), 2215(SM5), 2216(SM5), 2217(SM5), 2218(ST8), 2219(ST8), 2220(EM4), 2221(ST8), 2222(ST8), 2223 (ST8), 2224(HY10), 2225(HY10), 2226(PO5), 2227(PO5), 2228(PO5), 2229(ST8), 2230(EM4), 2231(EM4), 2232(AT4)c, 2233(PL3)c, 2234(EM4)c, 2235(HY10)c, 2236(SM5)c, 2237(ST8)c, 2238(PO5)c, 2239(ST8), 2240 (PL3).

NOVEMBER: 2241(HY11), 2242(HY11), 2243(HY11), 2244(HY11), 2245(HY11), 2246(SA6), 2247(SA6), 2248 (SA6), 2249(SA6), 2250(SA6), 2251(SA6), 2252(SA6), 2253(SA6), 2254(SA6), 2255(SA6), 2256(ST9), 2257(ST9), 2258(ST9), 2259(ST9), 2260(HY11), 2261(ST9)c, 2262(ST9), 2263(HY11), 2264(ST9), 2265(HY11), 2266(SA6), 2267(SA6), 2268(SA6), 2269(HY11)c, 2270(ST9).

DECEMBER: 2271(HY12)c, 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278 (HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), 2285(PO6), 2286(PO6), 2287 (PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296 (SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322 (ST10), 2323(HW4)c, 2324(CP2)c, 2325(SM6)c, 2326(WW4)c, 2327(IR4)c, 2328(PO6)c, 2329(ST10)c, 2330 (CP2).

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JANUARY: 2331(EM1), 2332(EM1), 2333(EM1), 2334(EM1), 2335(HY1), 2336(HY1), 2337(EM1), 2338(EM1), 2339(HY1), 2340(HY1), 2341(SA1), 2342(EM1), 2343(SA1), 2344(ST1), 2345(ST1), 2346(ST1), 2347(ST1), 2348(EM1)c, 2349(HY1)c, 2350(ST1), 2351(ST1), 2352(SA1)c, 2353(ST1)c, 2354(ST1).

FEBRUARY: 2355(CO1), 2356(CO1), 2357(CO1), 2358(CO1), 2359(CO1), 2360(CO1), 2361(PO1), 2362(HY2), 2363(ST2), 2364(HY2), 2365(SU1), 2366(HY2), 2367(SU1), 2368(SM1), 2369(HY2), 2370(SU1), 2371(HY2), 2372(PO1), 2373(SM1), 2374(HY2), 2375(PO1), 2376(HY2), 2377(CO1)c, 2378(SU1), 2379(SU1), 2380(SU1), 2381(HY2)c, 2382(ST2), 2383(SU1), 2384(ST2), 2385(SU1)c, 2386(SU1), 2387(SU1), 2388(SU1), 2389(SM1), 2390(ST2)c, 2391(SM1)c, 2392(PO1)c.

MARCH: 2393(IR1), 2394(IR1), 2395(IR1), 2396(IR1), 2397(IR1), 2398(IR1), 2399(IR1), 2400(IR1), 2401(IR1), 2402(IR1), 2403(IR1), 2404(IR1), 2405(IR1), 2406(IR1), 2407(SA2), 2408(SA2), 2409(HY3), 2410(ST3), 2411 (SA2), 2412(HW1), 2413(WW1), 2414(WW1), 2415(HY3), 2416(HW1), 2417(HW3), 2418(HW1)c, 2419(WW1)c, 2420(WW1), 2421(WW1), 2422(WW1), 2423(WW1), 2424(SA2), 2425(SA2)c, 2426(HY3)c, 2427(ST3)c.

APRIL: 2428(ST4), 2429(HY4), 2430(PO2), 2431(SM2), 2432(PO2), 2433(ST4), 2434(EM2), 2435(PO2), 2436 (ST4), 2437(ST4), 2438(HY4), 2439(EM2), 2440(EM2), 2441(ST4), 2442(SM2), 2443(HY4), 2444(ST4), 2445 (EM2), 2446(ST4), 2447(EM2), 2448(SM2), 2449(HY4), 2450(ST4), 2451(HY4), 2452(HY4), 2453(EM2), 2454 (EM2), 2455(EM2)c, 2456(HY4)c, 2457(PO2)c, 2458(ST4)c, 2459(SM2)c.

MAY: 2460(AT1), 2461(ST5), 2462(AT1), 2463(AT1), 2464(CP1), 2465(CP1), 2466(AT1), 2467(AT1), 2468(SA3), 2469(HY5), 2470(ST5), 2471(SA3), 2472(SA3), 2473(ST5), 2474(SA3), 2475(ST5), 2476(SA3), 2477(ST5), 2478 (HY5), 2479(SA3), 2480(ST5), 2481(SA3), 2482(CO2), 2483(CO2), 2484(HY5), 2485(HY5), 2486(AT1)c, 2487 (CP1)c, 2488(CO2)c, 2489(HY5)c, 2490(SA3)c, 2491(ST5)c, 2492(CP1), 2493(CO2).

JUNE: 2494(IR2), 2495(IR2), 2496(ST6), 2497(EM3), 2498(EM3), 2499(EM3), 2500(EM3), 2501(SM3), 2502 (EM3), 2503(PO3), 2504(WW2), 2505(EM3), 2506(HY6), 2507(WW2), 2508(PO3), 2509(ST6), 2510(EM3), 2511 (EM3), 2512(ST6), 2513(HW2), 2514(HY6), 2515(PO3), 2516(EM3), 2517(WW2), 2518(WW2), 2519(EM3), 2520 (PO3), 2521(HY6), 2522(SM3), 2523(ST6), 2524(HY6), 2525(HY6), 2526(HY6), 2527(IR2), 2528(ST6), 2529 (HW2), 2530(IR2), 2531(HY6), 2532(EM3)c, 2533(HW2)c, 2534(WW2), 2535(HY6)c, 2536(IR2)c, 2537(PO3)c, 2538(SM3)c, 2539(ST6)c, 2540(WW2)c.

JULY: 2541(ST7), 2542(ST7), 2543(SA4), 2544(ST7), 2545(ST7), 2546(HY7), 2547(ST7), 2548(SU2), 2549(SA4), 2550(SU2), 2551(HY7), 2552(ST7), 2553(SU2), 2554(SA4), 2555(ST7), 2556(SA4), 2557(SA4), 2558(SA4), 2559(ST7), 2560(SU2)c, 2561(SA4)c, 2562(HY7)c, 2563(ST7)c.

AUGUST: 2564(SM4), 2565(EM4), 2566(ST8), 2567(EM4), 2568(PO4), 2569(PO4), 2570(HY8), 2571(EM4), 2572(EM4), 2573(EM4), 2574(SM4), 2575(EM4), 2576(EM4), 2577(HY8), 2578(EM4), 2579(PO4), 2580 (EM4), 2581(ST8), 2582(ST8), 2583(EM4)c, 2584(PO4)c, 2585(ST8)c, 2586(SM4)c, 2587(HY8)c.

SEPTEMBER: 2588(IR3), 2589(IR3), 2590(WW3), 2591(IR3), 2592(HW3), 2593(IR3), 2594(IR3), 2595(IR3), 2596 (HW3), 2597(WW3), 2598(IR3), 2599(WW3), 2600(WW3), 2601(WW3), 2602(WW3), 2603(WW3), 2604(HW3), 2605(SA5), 2606(WW3), 2607(SA5), 2608(ST9), 2609(SA5)c, 2610(IR3), 2611(WW3)c, 2612(ST9)c, 2613(IR3)c, 2614(HW3)c.

OCTOBER: 2615(EM5), 2616(EM5), 2617(ST10), 2618(SM5), 2619(EM5), 2620(EM5), 2621(ST10), 2622(EM5), 2623(SM5), 2624(EM5), 2625(SM5), 2626(SM5), 2627(EM5), 2628(EM5), 2629(ST10), 2630(ST10), 2631(PO5)c, 2632(EM5)c, 2633(ST10), 2634(ST10), 2635(ST10)c, 2636(SM5)c.

c. Discussion of several papers, grouped by divisions.

PART 2

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PART 2

Your attention is invited

NEWS OF THE SOIL MECHANICS AND FOUNDATIONS DIVISION OF ASCE

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

JOURNAL OF THE SOIL MECHANICS AND FOUNDATIONS DIVISION
PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

DIVISION ACTIVITIES
SOIL MECHANICS AND FOUNDATIONS DIVISION
Proceedings of the American Society of Civil Engineers

NEWS

October, 1960

COLORADO SHEAR CONFERENCE

The following letter has been received from Dr. J. O. Osterberg, Chairman of the Executive Committee.

Fellow Members of the Soil Mechanics and Foundation Division

The conference on Shear Strength of Soils at Boulder last June was a huge success. We had over 400 in attendance, and there were just as many seats occupied the last day as the first. Credit for the success is due to the hard work and excellent planning of the task committee which arranged the conference - Bill Turnbull, Chairman, Juul Hvorslev, Vice-Chairman, Reg. Barron, Arthur Casagrande, Jack Hilf, Ralph Peck and Harry Seed. Thanks should go to them and many others who helped with the arrangements.

In response to your Chairman's request for comments and suggestions concerning this and future conferences, 58 replies were received. While it is impossible to convey to you fully the ideas presented in these replies, a summary will give an idea of what some of those in attendance thought. A great many volunteered their opinion that the conference was excellent, that the format, location, and arrangements were very fine.

The following suggestions were made for future conferences (number in parenthesis indicate number of commenters making suggestions):
Another conference on shear strength of cohesive soils (9)
Shear strength of sands and silts (1)
Consolidation and settlement of clay soils (12)
Application of laboratory test results to field problems (4)
Design and performance of earth dams (3)
Physico - chemical aspects of clay soils (3)
Force - deformation-time characteristics of soils (3)
Lateral earth pressures (3)
Dynamic properties of soils (2)
Stability analysis of clay soils (2)
Pore pressure measurements and their evaluation in relation to stability of earth dams (2)

Note.—No. 1960-33 is Part 2 of the copyrighted Journal of the Soil Mechanics and Foundations Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SM 5, October, 1960.

COLORADO SHEAR CONFERENCE

Soil testing techniques and equipment (2)
Stress-strain relationship of soils (2)
Field measurements and case histories (2)
Compacted soils (1)
Engineering Geology (1)
Bearing capacity of footings and piles (1)
Consolidation testing (1)
Case histories of shear failures (1)
Limitations of total and effective stress analysis (1)

Other suggestions include: have another conference in two to three years (7); in 4 years (3); prefer panel discussion type of conference over formal presentation of papers (13); panels too large (4); too few discussions of papers by panel (2); should allow time for more questions from audience (3).

Your executive committee will consider the suggestions and comments in formulating plans for a future conference.

ACTIVITIES OF THE SOIL MECHANICS AND FOUNDATION DIVISION COLORADO SECTION OCTOBER 1959 TO JUNE 1960

No.	Date	Speaker	Title	Attendance
1	Oct. 7, 1959	S. W. Pressey S. W. Pressey & Sons Pueblo, Colorado	"Presenting of Acker Drilling Equipment"	19
2	Nov. 4, 1959	O. L. Rice Chief Dams Branch Bureau of Reclamation Denver, Colorado	"Notable Structures and their Adaptation to Foundation Conditions in Foreign Lands"	18
3	Dec. 2, 1959	D. C. Harrington, R. A. Bohman, Engineers Bureau of Public Roads Denver, Colorado	"Materials and Stabilization Problems in Highway Construction," emphasis on landslides	32
4	Jan. 6, 1960	Guy F. Tabor Project Engineer Woodward-Clyde and Associates Denver, Colorado	"Discussion of Design Construction Control of Embankment by Overloading on the New Jersey Turnpike Extension"	23
5	Feb. 3, 1960	Lou Boduroff Boduroff and Meheen, Consulting Engineers Denver, Colorado	"The Influence of Differential Settlement on Statistically Indeterminate Structures in Evaluating Allowable Soil Pressures"	

ACTIVITIES OF THE SOIL MECHANICS AND FOUNDATION DIVISION
COLORADO SECTION
OCTOBER 1959 TO JUNE 1960

Continued

No.	Date	Speaker	Title	Attendance
6	Mar. 2, 1960	A. B. Reeves Tipton & Kalmbach, Consulting Engineers Denver, Colorado	"Unusual Problems Encountered in the Construction of the Roberts Tunnel," for the Denver Board of Water Commissioners	32
7	Apr. 6, 1960	R. O. Dickenson Public Service Company of Colorado Denver, Colorado	"Foundation on Structural Fill for 66,000 KW Power Plant at Cameo, Colorado	20
8	May 4, 1960	H. G. Arthur Designing Engineer Dams Branch Bureau of Reclamation Denver, Colorado	"Design Problems at Diguillen Dam, Chile"	20
9	June 13-17, 1960	- - - - - - - - -	Research Conference on Shear Strength of Cohesive Soils	Approx. 400 Registered

All meetings were held in the Stearns-Roger Building, Denver, Colorado, with exception of the Shear Conference which was held on the campus of the University of Colorado, Boulder, Colorado.

NEWS FROM THE ILLINOIS SECTION

The Illinois Section of the American Society of Civil Engineers has recently formed a Soil Mechanics and Foundations Division. Following is a brief account of the first meeting of this section.

The first meeting of the Soil Mechanics and Foundations Division of the Illinois Section, A.S.C.E. convened on June 29, 1960 with Dr. I. A. DuBose as Chairman, M. Salisbury as Vice-Chairman and D. Novick as Secretary-Treasurer. In attendance were about sixty engineers interested in the advancement of this rapidly growing field.

The "Research Conference on Shear Strength of Cohesive Soils" recently held at Boulder, Colorado, was discussed by those at the meeting who had attended the Conference - Drs. E. Vey, J. O. Osterberg, J. Schmertmann, E. T. Selig, and Messrs. D. Novick, C. W. Newlin, W. H. Perloff, H. Wall and B. Schimming.

For those who missed the Conference, this meeting provided an excellent opportunity to keep abrest of the latest advances in Soils Research.

The Soil Mechanics and Foundation Division of the Illinois Section of the A.S.C.E. held their second meeting on July 28, 1960 with thirty-two members and guests present. Mr. James F. Shook of the A.A.S.H.O. Test Road spoke about the "Statistical Approach to the Analysis of Soil Compaction Data." Following Mr. Shook's talk, Mr. Ronald Hudson introduced a movie illustrating the research work being done at the Test Road.

Mr. R. Alan Berggren reports the successful start of this newly formed division group and invites those interested to attend when they can. A third meeting was held on September 14, 1960 at Toffenetti's Restaurant, 65 W. Monroe Street, Chicago, with cocktails at 5:30, dinner at 6:00 and the meeting at 7:00. The discussion topic was "pile load tests."

PROCEEDINGS AVAILABLE

Proceedings of the Eighth Annual Northwestern Section Conference on Soil Mechanics and Foundation Engineering are available to interested persons from the Center for Continuation Study, University of Minnesota, Minneapolis 14, Minnesota.

The conference sponsored by the Northwestern Section, was held on April 14, 1960, and featured several papers on the general subject of Pile foundations.

NOMENCLATURE AND DEFINITIONS

Attention of Division members is called to the availability of a report prepared by a joint ASCE-ASTM Committee, entitled "Definitions of Terms and Symbols Relating to Soil Mechanics." This was published in the ASTM Standards for 1958, Part 4, under Designation D653-58T.

The use of these symbols will help to promote mutual understanding of material presented in reports and technical papers.

Mr. W. G. Holtz reports that in reading papers for preprinting in connection with the recent Shear Conference in Colorado, he estimated 25% of his time was spent thumbing back and forth between the texts and the authors' lists of terms of definitions and symbols. The use of so many different expressions makes reading and understanding difficult and, in some instances, may even lead to misunderstanding.

FIFTH INTERNATIONAL CONFERENCE ON SOIL MECHANICS AND FOUNDATION ENGINEERING PARIS 17 - 22 JULY 1961

The Organizing Committee for the Fifth Conference has sent Bulletin No. 1 to each member of the United States National Committee. Bulletin No. 2, giving full details of the program, fees, hotels, tours and excursions will be sent in December 1960 to all members of the International Society who have sent in a Preliminary Application Form. Following is a resume of certain information in Bulletin No. 1 which is believed to be of general interest:

July 1961	
- 9	Large Dams Congress, Rome - International Commission on Large Dams
	Fifth International Congress of Soil Mechanics and Foundation Engineering
10 - 14	a) Study tour south-east France, starting point Nice b) Study tour south-east France, starting point Paris
17 - 22	Fifth International Congress on Soil Mechanics and Foundation Engineering - Paris
24 - 30	c) Study tour French Alps, Basing of the Durance and Rhone Rivers, starting point Paris
24 - 27	d) Study tour Basin of the Seine, Dunkirk, starting point Paris

The fee for registration at the Conference will be about $51 for delegates and about $21 for ladies. The $51 includes the cost of the three volumes of Proceedings of the Conference. Additional sets of Proceedings may be procured by National Committee members at a cost of approximately $25. The first two volumes will be published in advance of the Conference and will contain the papers, and the surveys of new developments in each technical division prepared by the General Reporters. Volume 3 will be published about six months after the Conference and will contain the discussions, visits and daily affairs of the Conference.

The papers of the Conference will be divided into the same seven categories as used for the Fourth Conference in London. Discussions at Technical Sessions will be led by a chairman and will be initiated by the general reporter for the particular session. Discussions will be by specialists designated in advance who have made a special study of the subject in question and by certain authors of papers and of written discussions. If time permits, the chairman will open the discussion to the general assembly. Discussions will be directed toward the following particular items:

Section 1 (Soil properties and their measurement.)

a) Scatter of soil mechanics tests
b) Pore-pressure dissipation
c) Visco-elastic properties of soils; the existence of a Bingham limit; flow

Section 2 (Techniques of field measurement and sampling.)

In-situ tests for determination of mechanical properties of soils. Comparison with laboratory results.

Section 3 (Foundations of structures.)

A - Shallow Foundations

a) Influences of the shape and the dimensions of the foundations.
b) Non-saturated soils. Expansion and shrinkage.

B- Piled Foundations

a) Determination of the bearing capacity of a foundation from penetrometer tests.

b) Pile groups; bearing capacity and settlement.

Section 4 (Roads, runways, and rail-tracks.)

Deformation of roadways.

Section 5 (Earth Pressure on structures and tunnels.)

Variation in the active and passive related to the deformations in the vicinity of the structure.

Section 6 (Earth dams; slopes, and open excavations.)

Stability of slopes: considered as a function of time; effect of engineering works.

Section 7 (Various questions)

Best laboratory-works relationship. Use of soils as a constructional material in housing projects.

Regarding papers from the United States, about fifty-five summaries of proposed papers were received. Unfortunately, as in the case of the Fourth Conference at London, space for only twenty-five papers was allocated to the U.S.A. by the Organizing Committee. We were able to have this limit raised to thirty and the papers have been submitted for publication in the Proceedings.

NOTE: This announcement has been repeated at the request of the Executive Committee.

INTERNATIONAL STUDY OF SOIL SAMPLING

An international group of soil sampling was established during the Fourth International Conference on Soil Mechanics and Foundation Engineering, London, 1957. The immediate objectives of the group are to gather information on soil sampling equipment and procedures currently used in various countries or regions, and to promote needed research. The ultimate purpose is to formulate detailed requirements for obtaining undisturbed samples of soils, to evaluate the standard penetration test, and to suggest more or less standard types and principal dimensions of soil sampling equipment insofar as this can be accomplished without impeding further research and development. The international group is planning to submit a preliminary report to the Fifty International Conference on Soil Mechanics and Foundation Engineering, Paris, 1961. National units of the International group may publish more detailed reports on their findings if so desired.

Two questionnaires have been prepared and distributed to a selected mailing list. One deals with undisturbed sampling and the other with dynamic penetration test methods. If you believe you may have something to contribute and did not receive the questionnaires, please contact:

Professor J. O. Osterberg
Technological Institute
Northwestern University
Evanston, Illinois

COMBINED INDEX TO ASCE PUBLICATIONS

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DECEMBER NEWSLETTER

Deadline date for arrival at this office of contributions for the December issue of the Newsletter: October 26, please.

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